



EUNOIA JUNIOR COLLEGE

JC2 Preliminary Examination 2020

General Certificate of Education Advanced Level

Higher 2

CANDIDATE
NAME

CLASS

INDEX NO.

MATHEMATICS

9758/02

16 September 2020

Paper 2 [100 marks]

3 hours

Candidates answer on the Question Paper

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name, civics group and question number on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

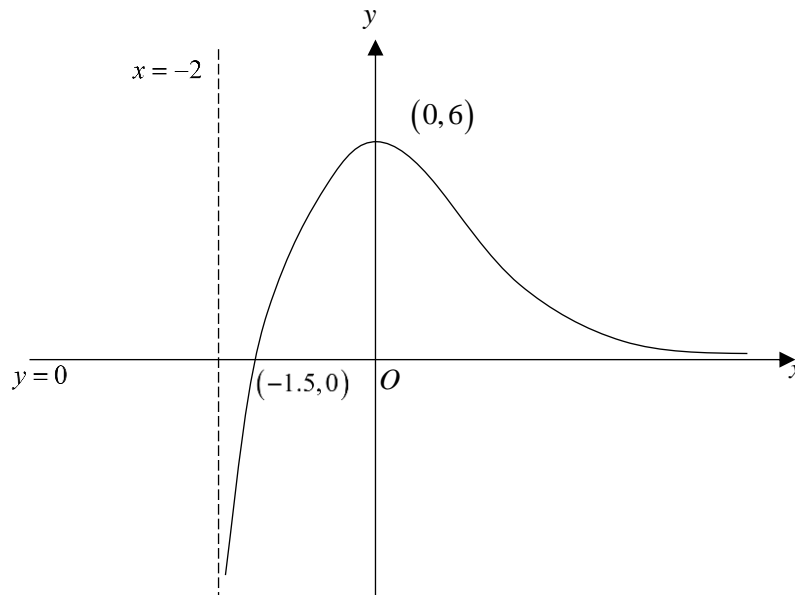
The number of marks is given in brackets [] at the end of each question or part question.

This document consists of **28** printed pages (including this cover page) and **2** blank page.

For markers' use:											
Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Total

Section A: Pure Mathematics [40 marks]

1



The diagram above shows the curve $y = f(x)$. The curve cuts the x -axis at the point $(-1.5, 0)$ and has a maximum point at $(0, 6)$. The curve also has vertical asymptote $x = -2$ and horizontal asymptote $y = 0$. Stating the equations of the asymptotes and coordinates of any turning points and points where the curves cross the axes, if it is possible to do so, sketch on separate diagrams, the curves

(i) $y = \frac{1}{2}f(1-x)$, [3]

(ii) $y = \frac{1}{f(x)}$. [3]

2 The curve C_1 has equation $y = \frac{3x^2 - 2x + 3}{x - 1}$. The curve C_2 has equation $(x - 1)^2 - \frac{(y - 4)^2}{b^2} = 1$, where $b > 0$.

(i) Sketch C_1 , stating the equations of any asymptotes and the coordinates of any turning points and points where the curve crosses the axes. [4]

(ii) Hence, find the range of values of b such that there is no point of intersection between C_1 and C_2 . Using the maximum value of b found, sketch C_2 on the same diagram as **part (i)**. [3]

3 A curve C has parametric equations

$$x = 1 + \cos \theta, \quad y = \sin \theta + \theta + 1, \quad \text{for } 0 < \theta < \frac{\pi}{2}.$$

- (i) The point P on C has parameter p . Show that the normal to C at P crosses the y -axis at a point Q with coordinates $(0, p + 1)$. [5]
- (ii) The normal to C at P crosses the x -axis at point R . Given that point S is the midpoint of QR , find a cartesian equation of the curve traced by S as p varies. [3]

4 Vectors \mathbf{a} and \mathbf{b} are such that the modulus of \mathbf{a} is 2 and \mathbf{b} is a unit vector perpendicular to \mathbf{a} .

- (i) State the value of $\mathbf{a} \cdot (2\mathbf{a} \times 3\mathbf{b})$ and explain your answer briefly. [1]
- (ii) Find the numerical area of the parallelogram with adjacent sides defined by $2\mathbf{a} + \mathbf{b}$ and $5\mathbf{a} - 7\mathbf{b}$. [3]

A vector \mathbf{c} is such that $\mathbf{b} \times \mathbf{c} = 2\mathbf{a} \times 3\mathbf{b}$.

- (iii) Show that $\mathbf{c} = \lambda \mathbf{b} - 6\mathbf{a}$, where λ is a constant. [2]
- (iv) State the geometrical meaning of $|\mathbf{b} \cdot \mathbf{c}|$ and find the possible values of λ if $|\mathbf{b} \cdot \mathbf{c}| = 5$. [2]

5 On 1 Jan 2019, Emma takes up a study loan of \$30000 from a bank that offers two repayment plans. Plan A charges a monthly instalment of \$600 at the beginning of every month and increases by \$50 for every subsequent month. There is no interest charged; instead, the bank charges a one-time administrative fee of \$2500 which is to be added to the study loan.

- (i) (a) Show that, after the n^{th} payment, she would have paid a total amount of $\$(25n^2 + 575n)$. [2]
- (b) How many payments will it take for Emma to fully repay her loan? [2]

Plan B charges a fixed monthly instalment of $\$x$ which is to be paid at the beginning of each month, starting from 1 Jan 2019. There is no interest charged for the first month only. Thereafter, interest is charged at the end of each month on the outstanding amount at 1% per month, (i.e. interest will be first charged on 28 February 2019.)

- (ii) (a) Show that the amount she owes after the fourth payment is $(1.01)^2(30000) - (1.01)^2(2x) - 1.01x - x$. [2]
- (b) Use the formula for the sum of a geometric progression to find an expression, in terms of n and x , the amount she owes after the n^{th} payment, for $n \geq 2$. [2]
- (c) If Emma intends to repay the loan fully after 32 payments, find the least value of x . [2]
- (iii) Using the value of x in **part (ii)(c)**, determine which plan will be cheaper for Emma. [1]

Section B: Probability and Statistics [60 marks]

- 6 Nine letter tiles that spell the word PRINCIPAL are used.
- (i) Find the number of ways to arrange the nine tiles in a row such that the vowels are together. [2]
 - (ii) The tiles are shuffled and placed in a row randomly. Find the probability that the tiles spell PRINCIPAL. [2]
 - (iii) Eight of the nine tiles are taken and arranged in a circle. Find the number of ways that this can be done. [3]

- 7 For events A and B , it is given that $P(A \cap B') = 0.32$ and $P(B|A) = \frac{7}{23}$.
- (i) Show that $P(A \cap B) = 0.14$. [2]
 - (ii) Given further that $P(A|B) = \frac{7}{19}$, find $P(A \cup B)$. [2]

It is given that $P(C) = 0.5$, where C is an event such that events A and C are independent and B and C are independent.

- (iii) Find the greatest and least possible values of $P(A' \cap B' \cap C)$. [4]

- 8 The time spent per day, in minutes, by Eunice on social media Toktik is normally distributed with mean 20.2 and standard deviation 4.5. Due to an unfortunate event, Eunice is confined to home for two months. During this period, the time spent per day by Eunice on Toktik is recorded for 20 randomly chosen days, which averages 22.4 minutes.

It is assumed that the population variance remains unchanged throughout.

- (i) Test, at 1% significance level, if there has been a change in the average time spent per day by Eunice on Toktik. [4]
- (ii) Without carrying out another test, find the range of α for which a test at α % level of significance suggests an **increase** in the average time spent per day by Eunice on Toktik. [2]

Another set of 15 timings is taken. Using only this set of 15 timings, find the range of values within which the mean of this set of readings must lie such that the claim that there has been a change in the average time spent by Eunice on Toktik is supported at 5% level of significance. [3]

- 9 A computer chip generates a random number from the set $\{1, 2, 3, 4\}$. The probability distribution of the number generated, X , is given by

$$P(X = x) = \begin{cases} p & x = 1 \\ \frac{1-p}{3} & x = 2, 3, 4 \end{cases}$$

where $0 < p < 1$.

(i) Find $E(X)$ in terms of p and hence show that $\text{Var}(X) = \frac{2}{3}(1-p)(1+6p)$. [3]

(ii) X_1 and X_2 are two independent observations of X . Find $P(|X_1 - X_2| > 1)$ in terms of p . Hence, find the maximum possible value of this probability as p varies. [5]

It is given instead that $p = \frac{1}{2}$.

(iii) Find the probability that the sum of 50 independent observations of X exceeds 110. [3]

- 10 A manufacturer produces disposable masks. On average, 6% of the masks are faulty. The masks are sold in boxes of 25.

(i) State, in context, two assumptions needed for the number of faulty masks in a box to be well modelled by a binomial distribution. [2]

Assume now that the number of faulty masks in a box has a binomial distribution.

(ii) Find the probability that a box of 25 of these masks contains at least 3 faulty masks. [2]

(iii) Find the probability that a box of 25 of these masks contains at most 5 faulty masks given that it contains at least 3 faulty masks. [2]

(iv) A store bought 100 boxes of these masks. Find the probability that at least 3 faulty masks are found in no more than $\frac{1}{5}$ of the boxes. [2]

The manufacturer also produces bottles of hand sanitisers which are sold in boxes of n bottles each. The number of faulty bottles in a box is denoted by the random variable F . It is known that F follows a binomial distribution, with each bottle having a 10% probability of being faulty.

(v) Given that the most likely value of F is 4, find all possible values of n , without using a calculator. [4]

- 11** The masses, in pounds, of apples and pears have independent normal distributions with means and standard deviations as shown in the following table.

	Mean	Standard deviation
Apple	0.33	σ
Pear	0.45	0.08

- (i) The probability of a randomly chosen apple weighing less than t pounds and that weighing between t and 0.35 pounds is 0.34 each. Find the values of σ and t . [4]
- (ii) Three pears are chosen at random. Find the probability that one of them weighs less than the population mean mass and each of the other two pears has a mass within one standard deviation of the population mean mass. [2]

The apples cost \$2.20 per pound and the pears cost \$3.10 per pound.

- (iii) Find the probability that the total cost of 3 randomly chosen apples and 2 randomly chosen pears is more than \$5. [3]
- (iv) A fruit seller gives $p\%$ discount on the cost of a fruit pack containing 3 apples and 2 pears. Find the smallest value of p if the fruit seller wants to be at least 99% sure that the cost of the pack is below \$4.50 after discount. [4]