VICTORIA JUNIOR COLLEGE

JC 2 PRELIMINARY EXAMINATION 2022

CANDIDATE NAME	

CLASS		INDEX NUMBER	

H2 MATHEMATICS **9758/01**
Paper 1 **3 hours**

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)
Writing paper

READ THESE INSTRUCTIONS FIRST

Write your class and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

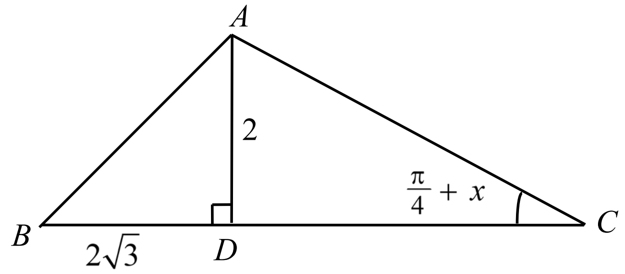
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This document consists of **21** printed pages and **3** blank pages.

1 A curve has equation $2x + y + 2 = (x + y)^2 + \dfrac{x^2}{1 + x^2}$.

Find the equation of the normal to the curve at the point $(0, 2)$. [5]

2 The diagram shows triangle ABC, where angle $ACD = \left(\frac{\pi}{4} + x\right)$ radians. Point D is on BC such that $AD = 2$ and $BD = 2\sqrt{3}$.



Show that if x is sufficiently small for x^3 and higher powers of x to be neglected, then

$$BC \approx k(1 + \sqrt{3} - 2x + 2x^2),$$

where k is a constant to be determined. [5]

3 **(a)** Find $\int (\ln x)^2 \, dx$. [3]

(b) Find $\int \frac{(\sin x + \cos x)^2}{\cos(2x) - 2x} \, dx$. [3]

4 A curve has equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \begin{cases} 2 - |x+1| & \text{for } -3 < x \leqslant 1, \\ 2 - 2(x-2)^2 & \text{for } 1 \leqslant x < 2. \end{cases}$$

(i) Sketch the curve for $-3 < x < 2$. [3]

(ii) Hence, solve the inequality $\mathrm{f}(x) \leqslant 0.1(x-1)^2$ for $-3 < x < 2$, leaving your answers in an exact form. [4]

5 **Do not use a calculator in answering this question.**

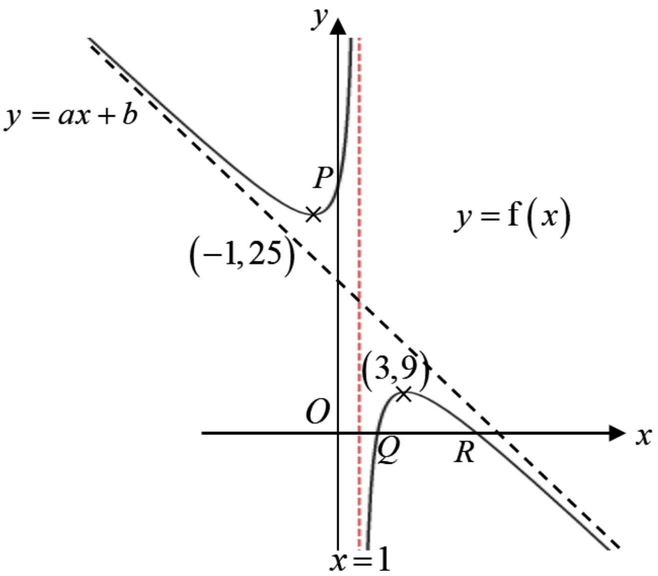
The complex number z satisfies the equation

$$z^2 - (4 + \mathrm{i})z + 2(\mathrm{i} - t) = 0,$$

where t is a real number. It is given that one root is of the form $k - k\mathrm{i}$, where k is real and positive. Find t and k, and the other root of the equation. [7]

[continued]

6



It is given that $f(x) = ax + b + \dfrac{c}{x-1}$, where a, b and c are constants. The diagram shows the curve with equation $y = f(x)$. The curve crosses the axes at points P, Q and R, and has stationary points at $(-1,\ 25)$ and $(3,\ 9)$.

Find the values of the constants a, b and c. [4]

It is now given that points P, Q and R have coordinates $(0, 27)$, $\left(\frac{3}{2}, 0\right)$ and $(9, 0)$ respectively.

Sketch the curve

(i) $y = \mathrm{f}(|x|)$, [2]

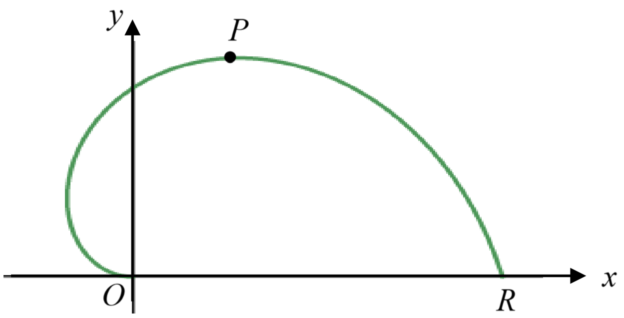
(ii) $y = \dfrac{1}{\mathrm{f}(x)}$, [3]

stating the equations of any asymptotes, the coordinates of any points where the curve crosses the axes and of any turning point(s).

7 The diagram below shows the curve C with parametric equations given by

$$x = -3\theta\cos 3\theta, \qquad y = 4\theta\sin 3\theta, \qquad \text{for } 0 \leqslant \theta \leqslant \frac{\pi}{3}.$$

Point P lies on C with parameter θ and C crosses the x-axis at the origin O and the point R.



(a) Find the area of the region bounded by C and the line $y = \frac{2\pi}{3} - \frac{2x}{3}$, giving your answer correct to 2 decimal places. [4]

(b) Use differentiation to find the maximum value of the area of triangle OPR as θ varies, proving that it is a maximum. [6]

8 **(a)** The sum of the first n terms of a sequence is given by $S_n = \frac{n^2 + 5n}{8}$. Show that the sequence follows an arithmetic progression with common difference d, where d is to be determined.

In a geometric progression, the first term is 100 and its common ratio is $3d$. Find the smallest value of k such that the sum of the first k terms of the arithmetic progression is greater than the sum of the first 30 terms of the geometric progression. [6]

(b) The first and second terms of a geometric sequence are $u_1 = a$ and $u_2 = a^2 - a$. If all the terms of the sequence are positive, find the set of values of a for which $\sum_{r=1}^{\infty} u_r$ converges. [2]

For this sequence, it is known that the sum of all the terms after the nth term is equal to the nth term. Find the value of a and hence the value of $\sum_{r=1}^{\infty} u_r$. [3]

9 The curve C has equation $\frac{1}{3}x^2 + y^2 - 2y = 0$.

(i) Sketch C. [2]

(ii) Use the substitution $x = p\sin\theta$ to show that

$$\int_0^p \sqrt{p^2 - x^2}\,\mathrm{d}x = \frac{p^2\pi}{4},$$

where p is a positive constant. [4]

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

(iii) The region R is bounded by C, the line $x = \sqrt{3}$ and the x-axis. Find the exact area of R. [3]

(iv) R is rotated completely about the y-axis. Find the exact volume of the solid obtained. [3]

(v) Describe a pair of transformations which transforms the graph of C onto the graph of $x^2 + y^2 = 1$. [2]

10 Workers are installing zip lines at an adventure park. The points (x, y, z) are defined relative to the entrance at $(0,0,0)$ on ground level, where units are in metres. The ticketing booth at $(100,100,1)$ and lockers at $(200,120,0)$ are also on ground level. Zip lines are laid in straight lines and the widths of zip lines can be neglected. The ground level of the park is modelled as a plane.

(i) Find a cartesian equation of the plane that models the ground level of the park. [2]

A zip line connects the points $P(300,120,30)$ and $Q(300,320,25)$, and is modelled as a segment of the line l. The façade of a building nearby can be modelled as part of the plane with equation $\mathbf{r}.\begin{pmatrix} 1 \\ -5 \\ 100 \end{pmatrix} = 0$. As a safety requirement, every point on the zip line must be at a distance of at least 10 metres away from the façade of the building.

(ii) Write down a vector equation of l. Hence, or otherwise, determine if the zip line passes the safety requirement. [4]

The workers need to install another zip line from Q to $R(127, 220, a)$, where $0 < a < 30$, and the angle PQR is given to be $60°$.

(iii) Find the value of a, leaving your answer to 3 decimal places. [3]

10 [Continued]

The façade of the building meets the ground level of the park at line m. A worker sets up a transmitter at point S on line m such that S is nearest to Q.

(iv) Find a vector equation of m and the distance from Q to S. [4]

11 A tank contains 500 litres of water in which 100 g of a poisonous chemical called Prokrastenate is dissolved. A solution containing 0.1 g of Prokrastenate per litre is pumped into the tank at a rate of 5 litres per minute, and the well-mixed solution is pumped out at the same rate. By letting x grams be the amount of Prokrastenate in the tank after t minutes, show that

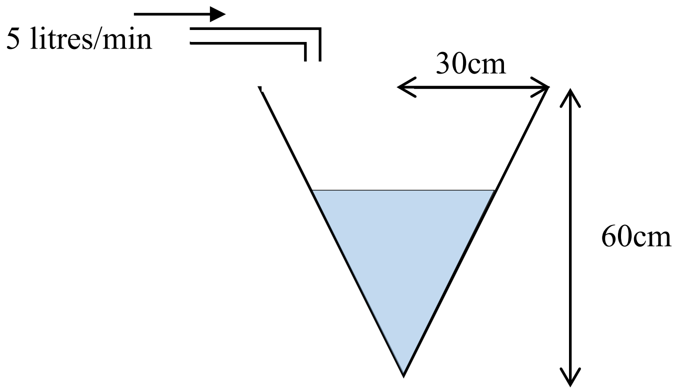
$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{k - x}{100},$$

where k is a constant to be determined. [2]

Find x in terms of t and find the value of t when $x = 75$. [5]

11 [Continued]

The well-mixed solution that is pumped out flows into an empty container, in the form of an open inverted cone with a height of 60 cm and base radius 30 cm, at the same rate (see diagram).



Given that 1 litre = 1000 cm^3,

(i) Show that the volume of the well-mixed solution in the container, V cm^3 can be expressed as $V = \dfrac{\pi h^3}{12}$, where h cm is the depth of the solution at that instant. [2]

(ii) Hence, or otherwise, find the rate of change of the depth of solution after 5 minutes. [4]

[The volume of a cone of base radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$.]

BLANK PAGE

BLANK PAGE

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

BLANK PAGE