1 The number of units, $D(x)$ of a particular product that people are willing to purchase per week in city A at a price \$$x$ is given by the function $D(x)=\dfrac{40320}{\mathrm{g}(x)}$, where $\mathrm{g}(x)$ is a quadratic polynomial in x. The following table shows the number of units people are willing to purchase at different prices.

x	5	8	10
$D(x)$	384	224	168

Find the number of units of the product that people are willing to purchase at a price of \$18. [4]

2 It is given that $p_n=\ln\dfrac{1+x^n}{1+x^{n+1}}$, where $-1<x<1$ and n is a positive integer.

(i) Find $\sum_{n=1}^{N} p_n$, giving your answer in terms of N and x. [3]

(ii) Hence find the sum to infinity of the series in part **(i)** in terms of x. [2]

3 In the triangle ABC, $AB=1$, $AC=2$ and angle $ABC=\left(\dfrac{\pi}{2}-x\right)$ radians. Given that x is sufficiently small for x^3 and higher powers of x to be ignored, show that $BC\approx p+qx+rx^2$, where p, q, r are constants to be determined in exact form. [5]

4 A part of a hyperbola has equation given by $\mathrm{f}(x)=\sqrt{\dfrac{(x+5)^2}{36}-1}$, $x\in D$, where $D\subseteq\mathbb{R}$.

(i) State the largest possible set D. [1]

(ii) State the equations of the asymptotes of $y=\mathrm{f}(x)$. [1]

(iii) Sketch the graph of $y=\mathrm{f}(x)$ for D in part **(i)**, showing clearly all the features of the curve. [2]

(iv) On separate diagrams, sketch the graphs of $y=\dfrac{1}{\mathrm{f}(x)}$ and $y=\mathrm{f}'(x)$, showing clearly all the features of the curves. [4]

5 Referred to the origin O, points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively. Point C lies on OB produced such that $\overrightarrow{OC} = \lambda\overrightarrow{OB}$ where $\lambda > 1$. Point D is such that $OCDA$ is a parallelogram. Point M lies on AD, between A and D, such that $AM : MD = 1 : 2$. Point N lies on OC, between O and C, such that $ON : NC = 4 : 3$.

(i) Find the position vectors of M and N, in terms of $\mathbf{a}$, $\mathbf{b}$ and λ. [2]

(ii) Show that the area of triangle OMD is $k\lambda\left|\mathbf{a}\times\mathbf{b}\right|$, where k is a constant to be determined. [4]

(iii) The vector $\mathbf{p}$ is a unit vector in the direction of $\overrightarrow{OD}$. Give a geometrical meaning of $\left|\mathbf{p}.\mathbf{a}\right|$. [1]

6 A curve C is defined by the parametric equations

$$x = 25\sin^2 t,\ \ y = 2\cos t$$

where $0 \le t \le \pi$.

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(ii) The tangent to C at the point where $t = \dfrac{\pi}{4}$ cuts the x-axis at P and the y-axis at Q, find the exact area of the triangle OPQ, where O is the origin. [4]

(iii) State the equation of the normal to C where the normal is parallel to the x-axis. [1]

7 **(a)** **(i)** Find $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(2\mathrm{e}^{\cos\frac{x}{2}}\right)$. [1]

(ii) Hence find $\displaystyle\int \frac{1}{2}\sin x\ \mathrm{e}^{\cos\frac{x}{2}}\ \mathrm{d}x$. [4]

(b) Using the substitution $u = 1 - \mathrm{e}^x$, find $\displaystyle\int \frac{1}{1-\mathrm{e}^x}\ \mathrm{d}x$. [4]

8 The function f is defined by $\mathrm{f}: x \mapsto \sqrt{a^2 - \dfrac{(x-a)^2}{4}}$ for $x \in \mathbb{R}$, $-a \le x \le 3a$, where a is a positive constant.

(i) Sketch the graph of $y = \mathrm{f}(x)$, giving the coordinates of any stationary points and the points where the graph meets the axes. [2]

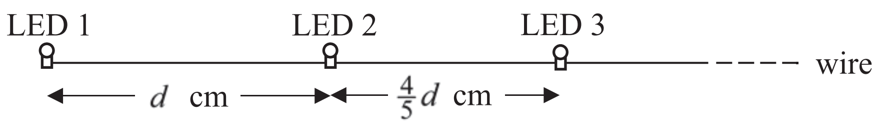
(ii) If the domain of f is further restricted to $-a \le x \le k$, state the greatest value of k for which the function f^{-1} exists. [1]

(iii) Using the restricted domain found in part **(ii)**, find f^{-1} in similar form. [3]

The function g is defined by $\mathrm{g}(x) = \mathrm{f}\left(\dfrac{3}{2}x\right)$ for $x \in \mathbb{R}$, $-\dfrac{2}{3}a \le x \le 2a$.

(iv) Explain why the composite function gf exists and find the range of gf. [3]

9 A company produces festive decorative Light Emitting Diode (LED) string lights, where micro LEDs are placed at intervals along a thin wire. In a particular design, the first LED (LED 1) is placed on one end of a wire with the second LED (LED 2) placed at a distance of d cm from LED 1, and each subsequent LED is placed at a distance $\frac{4}{5}$ times the preceding distance as shown.



(i) If the distance between LED 8 and LED 9 is 56.2 cm, find the value of d correct to 1 decimal place. [2]

(ii) Find the theoretical maximum length of the wire, giving your answer in centimetres correct to 1 decimal place. [2]

The LEDs consisting of three colours red, orange and yellow, are arranged in that order in a repeated manner, that is, LED 1 is red, LED 2 is orange, LED 3 is yellow, LED 4 is red, LED 5 is orange, LED 6 is yellow, and so on.

(iii) Find the colour of the LED nearest to a point on the wire 12.9 m from LED 1. [3]

(iv) If the minimum distance between any two consecutive LEDs is 1 cm so that they can be mounted on the wire, find the colour of the last LED. [3]

10 **(a)** Solve the simultaneous equations $v + \mathrm{i}u = 2$ and $av - 2u = 3\mathrm{i}$, where a is a real constant. Simplify your answers to cartesian form $x + \mathrm{i}y$, where x and y are in terms of a. [4]

(b) It is given that $(x + k)$ is a factor of the equation,

$$bx^3 + (12b + \mathrm{i})x^2 + (b + 12\mathrm{i})x + 12b = 0,$$

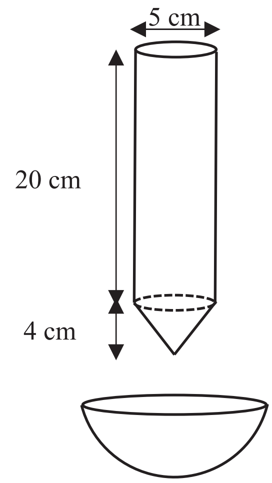
where k and b are non-zero real constants.

(i) Find the value of k. [2]

(ii) Show that the roots of the equation $bx^2 + \mathrm{i}x + b = 0$ are purely imaginary. [2]

(iii) Hence express $\mathrm{f}(x) = bx^3 + (12b + \mathrm{i})x^2 + (b + 12\mathrm{i})x + 12b$ as a product of three linear factors, leaving your answers in terms of b. [2]

11 A container is made up of a cylinder and an inverted right circular cone as shown in the diagram below. The height and the diameter of the cylinder are 20 cm and 5 cm respectively. The height of the cone is 4 cm. An external device ensures liquid flows out through a small hole at the vertex of the cone into a bowl below at a constant rate of 18 cm^3 per minute. The depth of the liquid and the radius of the liquid surface area in the container at the time t minutes are x cm and r cm respectively. The container is full of liquid initially.

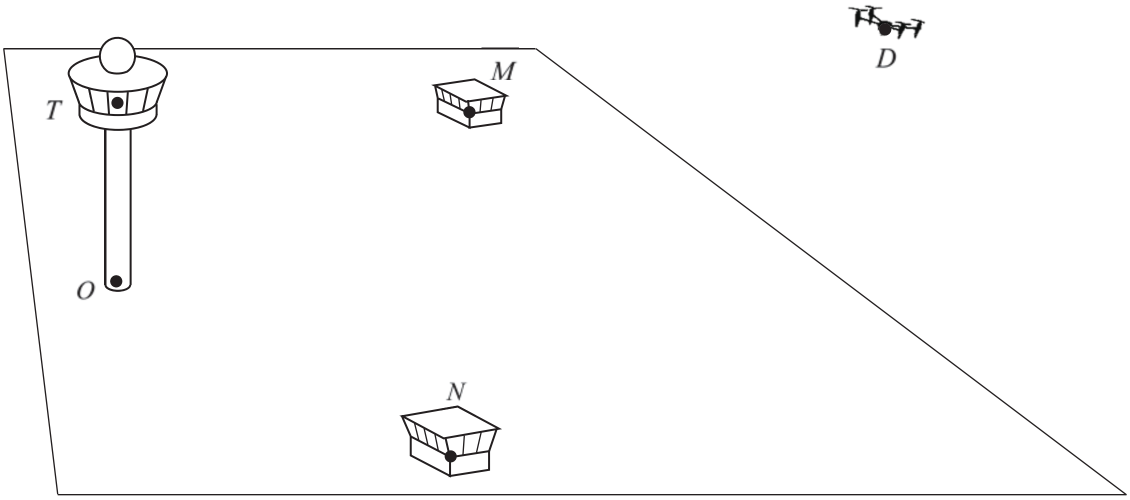


(a) When $x > 4$, find the rate of change of the depth of the liquid in the container. [4]

(b) Find the rate of change of r when $x = 2$. [5]

(c) The bowl as shown in the diagram in part **(a)** is generated by rotating part of the curve $\frac{x^2}{225} + \frac{y^2}{100} = 1$ which is below the x-axis through π radians about the y-axis. Assuming the bowl has negligible thickness, find the volume of the empty space in the bowl when the liquid has completely flowed from the container into the bowl, giving your answer correct to 3 decimal places. [3]

12 At an airport, an air traffic control room T is located in a vertical air traffic control tower, 70 m above ground level. Let $O(0,0,0)$ be the foot of the air traffic control tower and all points (x,y,z) are defined relative to O where the units are in kilometres. Two observation posts at the points $M(0.8,0.6,0)$ and $N(0.4,-0.9,0)$ are located within the perimeters of the airport as shown.



An air traffic controller on duty at T spots an errant drone in the vicinity of the airport. The two observation posts at M and N are alerted immediately. A laser rangefinder at M directs a laser beam in the direction $\begin{pmatrix} 2 \\ 7 \\ -1 \end{pmatrix}$ at the errant drone to determine D, the position of the errant drone. The position D is confirmed using another laser beam from N, which passes through the point $(0.8,0.75, 0.3)$, directed at the errant drone.

(i) Show that D has coordinates $(0.56,-0.24,0.12)$. [4]

A Drone Catcher, an anti-drone drone which uses a net to trap and capture errant drones, is deployed instantly from O and flies in a straight line directly to Γ intercept the errant drone.

(ii) Find the acute angle between the flight path of the Drone Catcher and the horizontal ground. [2]

At the same time, a Jammer Gun, which emits a signal to jam the control signals of the errant drone, is fired at the errant drone. The Jammer Gun is located at a point G on the plane p containing the points T, M and N.

(iii) Show that the equation of p is $\mathbf{r} \cdot \begin{pmatrix} -10.5 \\ 2.8 \\ -96 \end{pmatrix} = -6.72$. [3]

It is also known that the Jammer Gun is at the foot of the perpendicular from the errant drone to plane p.

(iv) Find the coordinates of G. [3]

(v) Hence, or otherwise, find the distance GD in metres. [2]