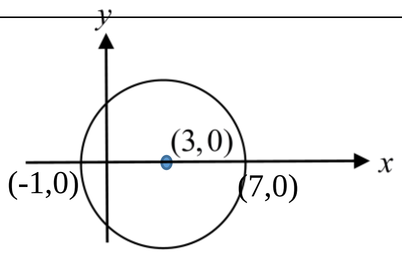
2020 RI H2 Mathematics Prelim Paper 1 Solutions

1 **[4]**	$16x^2+9y^2=144$ When $x=\sqrt{5}$: $y=\pm\sqrt{\dfrac{144-16(5)}{9}}=\pm\dfrac{8}{3}$ Differentiate equation with respect to x: $32x+18y\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)=0 \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{-32x}{18y}=-\dfrac{16x}{9y}$ $\dfrac{\mathrm{d}x}{\mathrm{d}t}=\dfrac{\mathrm{d}x}{\mathrm{d}y}\times\dfrac{\mathrm{d}y}{\mathrm{d}t}=-\dfrac{9y}{16x}\times 2=-\dfrac{9y}{8x}$ For $\dfrac{\mathrm{d}x}{\mathrm{d}t}>0$, x and y have different parity, and so the particle increases with respect to x at $(\sqrt{5},-\dfrac{8}{3})$. [**<u>Alternative 1:</u>** for position of particle : Since $\dfrac{\mathrm{d}y}{\mathrm{d}t}, x>0$, particle moves in anti-clockwise direction. Hence for $\dfrac{\mathrm{d}x}{\mathrm{d}t}>0$, y should be negative.] [**<u>Alternative 2:</u>** for position of particle : differentiate w.r.t t and get $32x\dfrac{\mathrm{d}x}{\mathrm{d}t}+18y\dfrac{\mathrm{d}y}{\mathrm{d}t}=0$. Since $\dfrac{\mathrm{d}x}{\mathrm{d}t},\dfrac{\mathrm{d}y}{\mathrm{d}t}, x>0$, y should be negative.] At $\left(\sqrt{5},-\dfrac{8}{3}\right)$, $\dfrac{\mathrm{d}x}{\mathrm{d}t}=-\dfrac{9}{8}\times\dfrac{-8}{3\sqrt{5}}=\dfrac{3}{\sqrt{5}}$ cms^{-1} At $\left(\sqrt{5},-\dfrac{8}{3}\right)$, its rate of increase is $\dfrac{3}{\sqrt{5}}$ cms^{-1} [Alternative for $\dfrac{\mathrm{d}x}{\mathrm{d}t}$, $32x\dfrac{\mathrm{d}x}{\mathrm{d}t}+18y\dfrac{\mathrm{d}y}{\mathrm{d}t}=0 \Rightarrow 32\sqrt{5}\dfrac{\mathrm{d}x}{\mathrm{d}t}+18\left(\dfrac{-8}{3}\right)(2)=0 \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}t}=\dfrac{3}{\sqrt{5}}$]

2 **[4]**	Let x, y and z be the amounts he invested into the 2%, 3% and 5% accounts respectively. $x+y+z=30000$ ------- (1) $0.02x+(1.03^2-1)y+0.05z=1423.50$ ------- (2) $x-z=1000$ ------- (3) From GC, solving the 3 equations, $x=8000$, $y=15000$, $z=7000$

3 **(i)** **[2]**	$y = ux \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = x\frac{\mathrm{d}u}{\mathrm{d}x} + u$ $(y-x)\left(\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{y}{x}\right) = y^2 + 2x^2$ $\Rightarrow (ux - x)\left(\left(x\frac{\mathrm{d}u}{\mathrm{d}x}+u\right) - u\right) = u^2x^2 + 2x^2$ $\Rightarrow (ux-x)\left(x\frac{\mathrm{d}u}{\mathrm{d}x}\right) = u^2x^2+2x^2$ $\Rightarrow (u-1)\left(\frac{\mathrm{d}u}{\mathrm{d}x}\right) = u^2+2 \qquad \because x>0$ $\Rightarrow \left(\frac{u-1}{u^2+2}\right)\left(\frac{\mathrm{d}u}{\mathrm{d}x}\right)=1$ $\Rightarrow \left(\frac{u}{u^2+2} - \frac{1}{u^2+2}\right)\left(\frac{\mathrm{d}u}{\mathrm{d}x}\right)=1$
(ii) **[3]**	$\left(\frac{u}{u^2+2} - \frac{1}{u^2+2}\right)\left(\frac{\mathrm{d}u}{\mathrm{d}x}\right)=1$ $\Rightarrow \int \frac{u}{u^2+2} - \frac{1}{u^2+2}\ \mathrm{d}u = \int 1\ \mathrm{d}x$ $\Rightarrow x = \frac{1}{2}\ln(u^2+2) - \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + C$ $x = \frac{1}{2}\ln\left(\left(\frac{y}{x}\right)^2 + 2\right) - \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{y}{\sqrt{2}x}\right) + C$

4 **(i)** **[2]**	$x^2 + y^2 - 6x = 7$ $(x-3)^2 + y^2 - 9 = 7$ $(x-3)^2 + y^2 = 4^2$ 
(ii) **[4]**	$x = 3 \Rightarrow 9 + y^2 - 6(3) = 7 \Rightarrow y^2 = 16 \Rightarrow y = \pm 4$ $x^2 + y^2 - 6x = 7 \Rightarrow (x-3)^2 + y^2 = 4^2 \quad \Rightarrow x = 3 \pm \sqrt{4^2 - y^2}$ Since $x \geq 3,\ x = 3 + \sqrt{4^2 - y^2}$. Volume of solid generated $= \pi\int_{-4}^{4} x^2 \mathrm{d}y - \pi(3)^2(2(4))$ $= \pi\int_{-4}^{4}\left(3 + \sqrt{4^2 - y^2}\right)^2 \mathrm{d}y - 72\pi$ $= \pi\int_{-4}^{4}\left(9 + 6\sqrt{4^2-y^2} + 16 - y^2\right)\ \mathrm{d}y - 72\pi$ $= \pi\left[25y - \frac{y^3}{3}\right]_{-4}^{4} + (6\pi)(2)\left[\frac{\pi}{4}(4^2)\right] - 72\pi$

	$=\pi\left(200-\frac{128}{3}\right)+48\pi^2-72\pi$ $=\frac{256}{3}\pi+48\pi^2$

5(i) **[1]**	$\int x\cos x^2\ \mathrm{d}x=\frac{1}{2}\sin x^2+c$
(ii) **[3]**	$\int x\cos 2x\ \mathrm{d}x=\frac{x}{2}\sin 2x-\frac{1}{2}\int\sin 2x$ $u=x \quad \frac{\mathrm{dv}}{\mathrm{d}x}=\cos 2x$ $\frac{\mathrm{d}u}{\mathrm{d}x}=1 \quad v=\frac{1}{2}\sin 2x$ $\int x\cos 2x\ \mathrm{d}x=\frac{x}{2}\sin 2x+\frac{\cos 2x}{4}+c$
[3]	$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x\cos^2 x\mathrm{d}x=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x\left(\frac{\cos 2x+1}{2}\right)\mathrm{d}x$ $=\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x\cos 2x\mathrm{d}x+\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x\mathrm{d}x$ $=\frac{1}{2}\left[\frac{x}{2}\sin 2x+\frac{\cos 2x}{4}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}+\frac{1}{2}\left[\frac{x^2}{2}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ $=\frac{1}{2}\left[\left(0-\frac{1}{4}\right)-\left(\frac{\pi}{8}+0\right)\right]+\frac{1}{2}\left[\frac{1}{2}\left(\frac{\pi^2}{4}-\frac{\pi^2}{16}\right)\right]$ $=\frac{3\pi^2}{64}-\frac{1}{8}-\frac{\pi}{16}$

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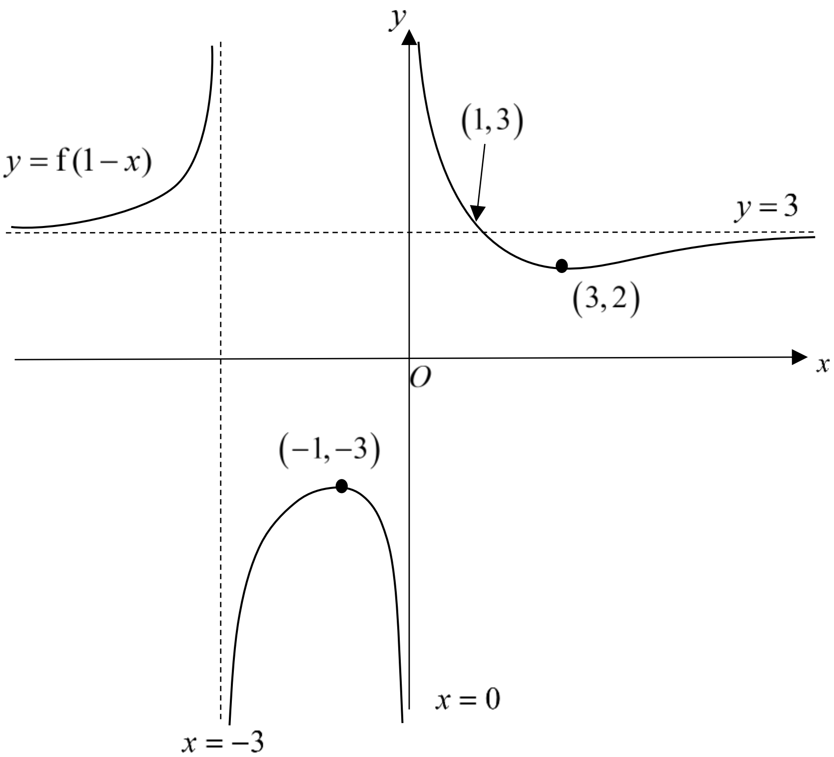
[2]</td><td>
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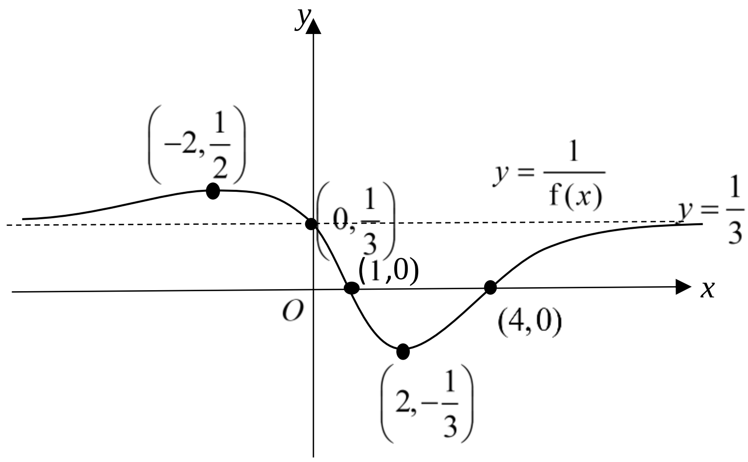
[3]</td><td>
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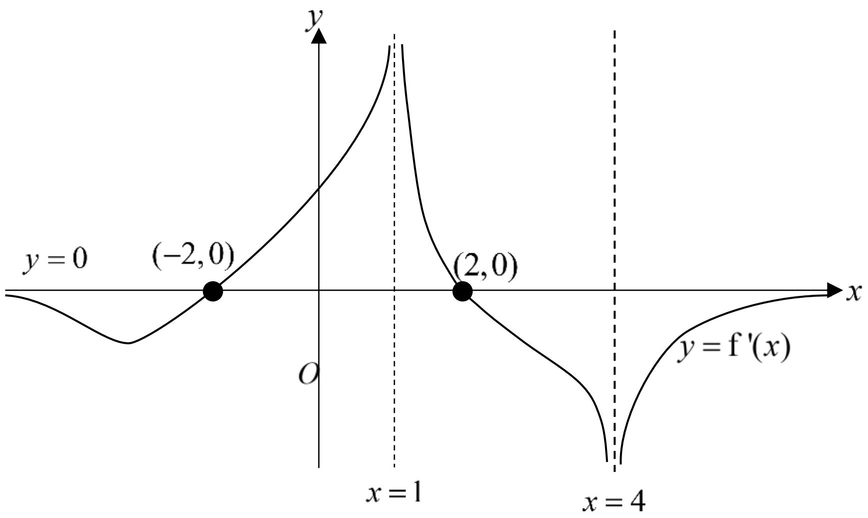
[3]</td><td>

Note: There is no way to label the y-intercept as there is no information on the gradient of the tangent when x = 0</td></tr>
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7 **[2]**	$\mathrm{f}(r+2)-\mathrm{f}(r)=\frac{2^{r+2}}{r}-\frac{2^{r}}{r-2}$ $=\frac{2^{r+2}(r-2)-2^{r}r}{r(r-2)}$ $=\frac{4r.2^{r}-8.2^{r}-r.2^{r}}{r(r-2)}$ $=\frac{(3r-8)2^{r}}{r(r-2)}$ (shown)
(i) **[4]**	$\sum_{r=3}^{n}\frac{(3r-8)2^{r}}{r(r-2)}=\sum_{r=3}^{n}\left(\mathrm{f}(r+2)-\mathrm{f}(r)\right)$ $=\left[\begin{array}{l}\mathrm{f}(5)-\mathrm{f}(3)\\+\mathrm{f}(6)-\mathrm{f}(4)\\+\mathrm{f}(7)-\mathrm{f}(5)\\+\mathrm{f}(8)-\mathrm{f}(6)\\\ldots\\+\mathrm{f}(n)-\mathrm{f}(n-2)\\+\mathrm{f}(n+1)-\mathrm{f}(n-1)\\+\mathrm{f}(n+2)-\mathrm{f}(n)\end{array}\right]$ $=\left[\mathrm{f}(n+1)+\mathrm{f}(n+2)-\mathrm{f}(3)-\mathrm{f}(4)\right]$ $=\left[\frac{2^{n+1}}{n-1}+\frac{2^{n+2}}{n}-8-\frac{16}{2}\right]$ $=\frac{2n.2^{n}+4n.2^{n}-4.2^{n}}{n(n-1)}-16$ $=\frac{(2n+4n-4)2^{n}}{n(n-1)}-16$ $=\frac{(3n-2)2^{n+1}}{n(n-1)}-16$
(ii) **[4]**	$\sum_{r=1}^{n}\frac{(3r-2)2^{r}}{r(r+2)}=\sum_{r=3}^{n+2}\frac{(3(r-2)-2)2^{r-2}}{(r-2)r}$ $=\sum_{r=3}^{n+2}\frac{(3r-8)2^{r-2}}{r(r-2)}$ $=\frac{1}{4}\sum_{r=3}^{n+2}\frac{(3r-8)2^{r}}{r(r-2)}$ $=\frac{1}{4}\left[\frac{(3(n+2)-2)2^{(n+2)+1}}{(n+2)(n+1)}-16\right]$ $=\frac{(3n+4)2^{n+1}}{(n+2)(n+1)}-4$ $\therefore A=4, B=2, C=-4$

8(a) (i) [3]	Let $z = x + iy$, $x, y \in \mathbb{R}$. Then $z^2 = 4i - 3 \quad \Rightarrow (x+iy)^2 = (x^2 - y^2) + 2ixy = 4i - 3$ $\Rightarrow \begin{cases} x^2 - y^2 = -3 \\ 2xy = 4 \end{cases}$ $\Rightarrow x^2 - \dfrac{4}{x^2} = -3$ $\Rightarrow x^4 + 3x^2 - 4 = (x^2+4)(x^2-1) = 0$ $\Rightarrow x = \pm 1$ When $x = 1, y = 2$. When $x = -1, y = -2$ Thus the roots are $1+2i$ and $-1-2i$.
(a) (ii) [3]	$z^4 + 6z^2 + 25 = 0 \quad ---(1)$ $\left(z^2+3\right)^2 + 16 = 0$ $z^2 + 3 = \pm 4i$ $z^2 = 4i - 3$ or $-4i - 3$ or $z^4 + 6z^2 + 25 = 0 \quad ---(1)$ $z^2 = \dfrac{-6 \pm \sqrt{36 - 4(25)}}{2}$ $= -3 \pm \dfrac{8i}{2} = -3 \pm 4i$ For $z^2 = 4i - 3$, $z = 1+2i,\ -1-2i$ Since (1) is an equation with real coefficients, the roots occur in conjugate pairs. Thus the roots of the equation $z^4 + 6z^2 + 25 = 0$ are $z = 1 \pm 2i,\ -1 \pm 2i$.
(b) [4]	$w = \dfrac{8-2i}{5+3i} = \dfrac{8-2i}{5+3i} \times \dfrac{5-3i}{5-3i}$ $= \dfrac{40 - 10i - 24i + 6i^2}{5^2 + 3^2}$ $= \dfrac{34 - 34i}{34} = 1 - i$ Thus $\lvert w \rvert = \sqrt{1^2 + 1^2} = \sqrt{2}$ and $\arg w = -\dfrac{\pi}{4}$ For w^n to be real, $w^n = \left(\sqrt{2}\right)^n \left[\cos\left(-\dfrac{n\pi}{4}\right) + i\sin\left(-\dfrac{n\pi}{4}\right)\right]$ is real. Hence $\sin\left(-\dfrac{n\pi}{4}\right) = 0 \Rightarrow \dfrac{n\pi}{4} = k\pi, k \in \mathbb{Z}$, and so $n = 4k,\ k \in \mathbb{Z}^+$ (since $n > 0$)

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(i)
[4]</td><td>

$x = t\ln t \Rightarrow \frac{dx}{dt} = t\left(\frac{1}{t}\right) + \ln t = 1 + \ln t,$

$y = \frac{4}{e^t} + e^t \Rightarrow \frac{dy}{dt} = -4e^{-t} + e^t = \frac{e^{2t}-4}{e^t},$

$\therefore \frac{dy}{dx} = \frac{e^{2t}-4}{e^t(1+\ln t)}$

Now, $x = 0 \Rightarrow t\ln t = 0 \Rightarrow t = 1\ (\because t > 0)$

$\Rightarrow y = \frac{4}{e} + e = \frac{4+e^2}{e}$ and $\frac{dy}{dx} = \frac{e^2-4}{e}$

Equation of normal at $P\ (0, \frac{4+e^2}{e})$:

$y - \frac{4+e^2}{e} = -\frac{e}{e^2-4}x \Rightarrow y = \frac{e}{4-e^2}x + \frac{4+e^2}{e}$

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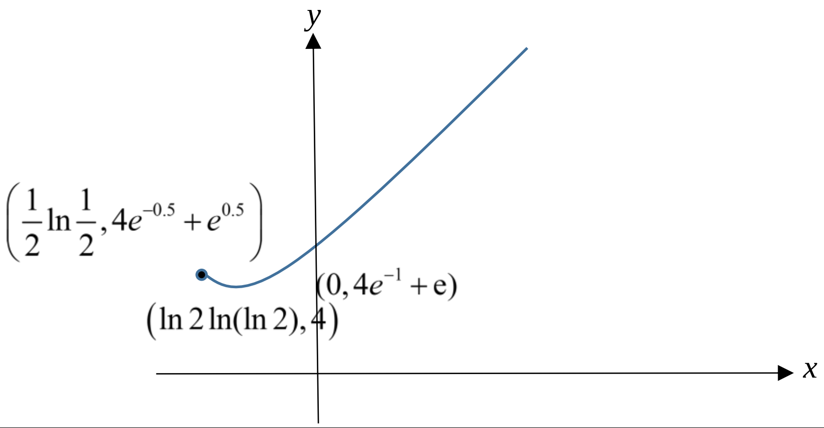
$\frac{dy}{dx} = \frac{e^{2t}-4}{e^t(1+\ln t)} = 0 \Rightarrow e^{2t} - 4 = 0 \Rightarrow t = \ln 2$

Min occurs at $x = \ln 2(\ln(\ln 2))$, $y = \frac{4}{e^{\ln 2}} + e^{\ln 2} = 4$



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[4]</td><td>

Area

$= \int_{0.5\ln 0.5}^{0} \left(\frac{e}{4-e^2}x + \frac{4+e^2}{e}\right) - y\,dx$

$= \int_{0.5\ln 0.5}^{0} \left(\frac{e}{4-e^2}x + \frac{4+e^2}{e}\right) dx - \int_{\frac{1}{2}}^{1} \left(\frac{4}{e^t} + e^t\right)(1+\ln t)\ dt$

$= 0.0943$ (3s.f.)

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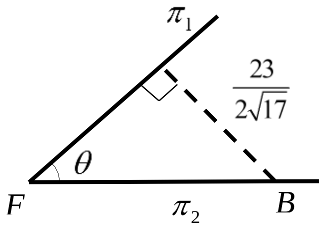
10 (i) [3]	$l:\mathbf{r}=\begin{pmatrix}1\\-1\\2\end{pmatrix}+\lambda\begin{pmatrix}2\\0\\-3\end{pmatrix},\ \lambda\in\mathbb{R}$ Let C be a point on l such that $\overrightarrow{OC}=\begin{pmatrix}1\\-1\\2\end{pmatrix}$. $\overrightarrow{AC}=\begin{pmatrix}1\\-1\\2\end{pmatrix}-\begin{pmatrix}1\\2\\-1\end{pmatrix}=\begin{pmatrix}0\\-3\\3\end{pmatrix}$ $\mathbf{n}_1=\begin{pmatrix}0\\-1\\1\end{pmatrix}\times\begin{pmatrix}2\\0\\-3\end{pmatrix}=\begin{pmatrix}3\\2\\2\end{pmatrix}$ $\pi_1:\mathbf{r}.\begin{pmatrix}3\\2\\2\end{pmatrix}=\begin{pmatrix}1\\-1\\2\end{pmatrix}.\begin{pmatrix}3\\2\\2\end{pmatrix}=3-2+4=5 \quad\Rightarrow \mathbf{r}.\begin{pmatrix}3\\2\\2\end{pmatrix}=5$
(ii) [3]	$\overrightarrow{OF}=\begin{pmatrix}1\\-1\\2\end{pmatrix}+\lambda\begin{pmatrix}2\\0\\-3\end{pmatrix}$, for some $\lambda\in\mathbb{R}$ $\overrightarrow{BF}=\overrightarrow{OF}-\overrightarrow{OB}=\begin{pmatrix}6.5+2\lambda\\-4\\-3\lambda\end{pmatrix}$ $\overrightarrow{BF}.\begin{pmatrix}2\\0\\-3\end{pmatrix}=0 \quad\Rightarrow 13+4\lambda+9\lambda=0$ $\lambda=-1$ $\therefore \overrightarrow{BF}=\begin{pmatrix}6.5+2(-1)\\-4\\-3(-1)\end{pmatrix}=\begin{pmatrix}4.5\\-4\\3\end{pmatrix}$ (shown)
(iii) [2]	Shortest distance from B to π_1 = length of projection of $\overrightarrow{BF}$ onto $\mathbf{n}_1$ $=\left\|\overrightarrow{BF}\cdot\hat{\mathbf{n}}_1\right\|=\dfrac{\begin{pmatrix}4.5\\-4\\3\end{pmatrix}\cdot\begin{pmatrix}3\\2\\2\end{pmatrix}}{\sqrt{17}}=\dfrac{23}{2\sqrt{17}}$ units
(iv) [3]	Let θ be the acute angle between π_1 and π_2

$$\sin\theta = \frac{\frac{23}{2\sqrt{17}}}{BF}$$

$$= \frac{23}{2\sqrt{17}} \div \left|\overrightarrow{BF}\right|$$

$$\theta = \sin^{-1}\frac{23}{2\sqrt{17}\sqrt{4.5^2+4^2+3^2}} = 24.5° \text{ (1d.p)}$$



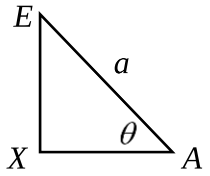
[Alternatively, $\mathbf{n}_2 = \overrightarrow{BF} \times \begin{pmatrix}2\\0\\-3\end{pmatrix} = \begin{pmatrix}4.5\\-4\\3\end{pmatrix} \times \begin{pmatrix}2\\0\\-3\end{pmatrix} = \begin{pmatrix}12\\19.5\\8\end{pmatrix}$

Let θ be the acute angle between π_1 and $\frac{23}{2\sqrt{17}}$

$$\theta = \cos^{-1}\frac{\begin{pmatrix}3\\2\\2\end{pmatrix}.\begin{pmatrix}12\\19.5\\8\end{pmatrix}}{\sqrt{17}\sqrt{588.25}} = \cos^{-1}\frac{91}{\sqrt{17}\sqrt{588.25}} = 24.5° \text{ (1d.p)}$$]

11 (i) [2]

By trigo ratio, $EX = a\sin\theta,\ AX = a\cos\theta$



$$V = a\left\{2\left[\frac{1}{2}(a\cos\theta)(a\sin\theta)\right] + a(a\sin\theta)\right\}$$

$$= a^3\sin\theta(1+\cos\theta)$$

[Alternatively, by area of trapezium $V = a\left\{\frac{1}{2}(a\sin\theta)\left[a+(a+2a\cos\theta)\right]\right\}$

$= a^3\sin\theta(1+\cos\theta)$]

(ii) [5]

$$\frac{dV}{d\theta} = a^3\left[\cos\theta(1+\cos\theta)+\sin\theta(-\sin\theta)\right]$$

$$= a^3\left[\cos\theta+\cos^2\theta-\sin^2\theta\right]$$

$$= a^3\left[\cos\theta+\cos 2\theta\right]$$

$$= 2a^3\cos\frac{3\theta}{2}\cos\frac{\theta}{2} \quad \text{(Factor Formulae)}$$

$$\frac{dV}{d\theta} = 0 \Rightarrow \cos\frac{3\theta}{2} = 0 \text{ or } \cos\frac{\theta}{2} = 0$$

$$\Rightarrow \frac{3\theta}{2} \text{ or } \frac{\theta}{2} = \frac{\pi}{2}$$

$$\Rightarrow \theta = \frac{\pi}{3} \text{ or } \pi \text{ (NA)}$$

Alternatively:

$$\frac{dV}{d\theta} = a^3\left[\cos\theta(1+\cos\theta)+\sin\theta(-\sin\theta)\right]$$

$$= a^3\left[\cos\theta+\cos^2\theta-\sin^2\theta\right]$$

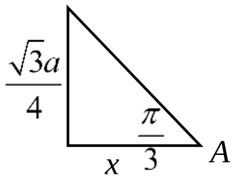
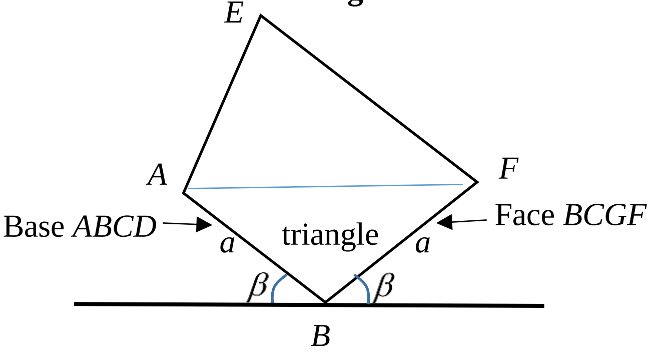
$$= a^3\left[\cos\theta+\cos^2\theta-\left(1-\cos^2\theta\right)\right]$$

$$= a^3\left[2\cos^2\theta+\cos\theta-1\right]$$

$$= a^3(2\cos\theta-1)(\cos\theta+1)$$

$$\frac{dV}{d\theta} = 0 \Rightarrow \cos\theta = \frac{1}{2} \text{ or } \cos\theta = -1$$

$$\Rightarrow \theta = \frac{\pi}{3} \text{ or } \pi \text{ (NA)}$$

	$\frac{dV}{d\theta} = a^3[\cos\theta + \cos 2\theta]$ At $\theta = \frac{\pi}{3}$, $\frac{d^2V}{d\theta^2} = a^3[-\sin\theta - 2\sin 2\theta]$ $= a^3\left[-\frac{\sqrt{3}}{2} - \sqrt{3}\right] = -\frac{3\sqrt{3}}{2}a^3 < 0$ Hence $\theta = \frac{\pi}{3}$ gives maximum value of V and $\max V = a^3\left(\frac{\sqrt{3}}{2}\right)\left(1+\frac{1}{2}\right) = \frac{3\sqrt{3}}{4}a^3$ cm^3
(iii) [3+2]	Half its height = $\frac{1}{2}\left(a\sin\frac{\pi}{3}\right) = \frac{\sqrt{3}a}{4}$.  $\therefore \tan\frac{\pi}{3} = \frac{\frac{\sqrt{3}a}{4}}{x} \Rightarrow \sqrt{3} = \frac{\sqrt{3}a}{4x} \Rightarrow x = \frac{a}{4}$ V(half its height) $= a\left[\left(\frac{\sqrt{3}a}{4}\right)a + \left(\frac{a}{4}\right)\left(\frac{\sqrt{3}a}{4}\right)\right] = \left(\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{16}\right)a^3 = \frac{5\sqrt{3}}{16}a^3$ cm^3 **Figure 3**  Cross-Sectional View Note that we can consider face *ABFE* as a possible cross-sectional view in Figure 3. Then $\angle ABF = \frac{2\pi}{3}$ and $AB = BF = a$, and so Area of triangle *ABF* = $\frac{1}{2}a^2\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{4}a^2$. Hence the volume of water that the container can hold at this position is at most $\frac{\sqrt{3}}{4}a^3 < V(\text{half the height})$, and so water will definitely flow out of the container before it reaches this position. So no, it is not possible. [Alternative explanation (for the case where θ may not be fixed): Note that area of triangle increases with θ and θ is acute. Hence max volume $< \frac{a^3}{2}\sin\frac{\pi}{2} = \frac{a^3}{2} < \frac{5\sqrt{3}}{16}a^3$.]

12 (i) [5]	$\frac{\mathrm{d}L}{\mathrm{d}t}=k(L_\infty-L)$, where k is the constant of proportionality. $\frac{\mathrm{d}L}{\mathrm{d}t}=k(L_\infty-L)$ $\frac{1}{L_\infty-L}\frac{\mathrm{d}L}{\mathrm{d}t}=k$ $\int\frac{1}{L_\infty-L}\,\mathrm{d}L=\int k\,\mathrm{d}t$ $-\ln(L_\infty-L)=kt+c$ $\because L_\infty-L>0$ $L_\infty-L=e^{-(kt+c)}$ $L_\infty-L=Ae^{-kt}$, A is a positive constant $L=L_\infty-Ae^{-kt}$, A is a positive constant Note that $L\neq L_\infty$ in this context. Also, $A>0$.
[3]	Since $L_\infty=419$ mm, $L=419-Ae^{-kt}$ When $t=1$, $L=219$ and thus $219=419-Ae^{-k}\Rightarrow Ae^{-k}=200$ --- (1) Also, $t=1$, $\frac{\mathrm{d}L}{\mathrm{d}t}=55\Rightarrow Ake^{-k}=55$ --- (2) Sub (1) into (2), $k=\frac{55}{200}=\frac{11}{40}$ (or 0.275) (alternatively, using $\frac{\mathrm{d}L}{\mathrm{d}t}=k(L_\infty-L)$, $55=k(419-219)$) Thus $A=200(e^{\frac{11}{40}})=263.31=263$ $L=419-263e^{-\frac{11}{40}t}$
(ii) [2]	When $L=300$, $300=419-263.31e^{-\frac{11}{40}t}\Rightarrow t=2.89$ years
(iii) [2]