NANYANG JUNIOR COLLEGE

JC2 PRELIMINARY EXAMINATION

Higher 2

Candidate Name	

CT Class	1	8			

Centre Number/ Index Number					/				

MATHEMATICS 9758/02

Paper 2

16th September 2019

3 Hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS

Write your name and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For examiner's use only	
Question number	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

This document consists of **5** printed pages.

NANYANG JUNIOR COLLEGE
Internal Examinations

[Turn Over

Section A: Pure Mathematics [40 marks]

1 **(i)** Show that $\dfrac{1}{(n-1)!}-\dfrac{3}{n!}+\dfrac{2}{(n+1)!}=\dfrac{An^2+Bn+C}{(n+1)!}$ where A, B and C are constants to be determined. [2]

(ii) Hence find $\displaystyle\sum_{n=1}^{N}\frac{n^2-2n-1}{5(n+1)!}$ in terms of N. [3]

(iii) Give a reason why the series $\displaystyle\sum_{n=1}^{\infty}\frac{n^2-2n-1}{5(n+1)!}$ converges and write down its value. [2]

2 The curve C has parametric equations $x=6t^2$, $y=\dfrac{2t}{\sqrt{1-t^2}}$, $0<t<1$.

(i) A line is tangent to the curve C at point A and passes through the origin O. Show that the line has equation $y=\dfrac{2}{3}x$. [4]

The region R is bounded by the curve and the tangent line in **(i)**.

(ii) Find the area of R. [3]

(iii) Write down the Cartesian equation of the curve C. [1]

(iv) Find the exact volume of the solid of revolution generated when R is rotated completely about the x-axis, giving your answer in the form $(a\ln b-c)\pi$, where constants a, b, c are to be determined. [4]

3 When a ball is dropped from a height of H m above the ground, it will rebound to a height of eH m where $0<e<1$. The height of each successive bounce will be e times of that of its previous height. It is also known that the time taken between successive bounce is given by $t=0.90305\sqrt{h}$ where h is the maximum height of the ball from the ground between these bounces. We can assume that there is negligible air resistance.
A ball is now dropped from a height of 10 m from the ground. Let t_n be the time between the n^{th} and $(n+1)^{\text{th}}$ bounce.

(i) Show that the total distance travelled by the ball just before the n^{th} bounce is $\dfrac{10(1+e-2e^n)}{1-e}$. [3]

(ii) Show that t_n is a geometric sequence. State the common ratio for this sequence. [3]

(iii) Find in terms of e the total distance the ball will travel and the time taken when it comes to rest. You may assume that between any two bounces, the time taken for the ball to reach its maximum height is the same as the time it takes to return to the ground. [3]

4 Referred to the origin, the points A and B have position vectors $-\mathbf{i}+3\mathbf{j}-2\mathbf{k}$ and $3\mathbf{i}+\mathbf{k}$ respectively. The plane π has equation $\mathbf{r}=\begin{pmatrix}5\\0\\0\end{pmatrix}+\lambda\begin{pmatrix}2\\0\\-1\end{pmatrix}+\mu\begin{pmatrix}1\\2\\a\end{pmatrix}$, and the line l has equation $\mathbf{r}=\begin{pmatrix}3\\0\\a\end{pmatrix}+t\begin{pmatrix}4a\\4\\1\end{pmatrix}$, where a is a constant and λ, μ, and t are parameters.

(i) Show that for all real values of a, l is parallel to π. [2]

(ii) Find the value of a such that l and π have common points. [2]

For the rest of the question, let $a=1$.

(iii) Find the projection of $\overrightarrow{AB}$ onto π. [3]

(iv) Let F be the foot of perpendicular from A to π. The point C lies on AF extended such that $\angle ABF=\angle CBF$. Find a cartesian equation of the plane that contains C and l. [3]

(v) Let D be a point on l. Find the largest possible value of the non-reflex angle $\angle ADC$. [2]

Section B: Probability and Statistics [60 marks]

5 This question is about arrangements of all nine letters in the word ADDRESSEE.

(i) Find the number of different arrangements of the nine letters. [1]

(ii) Find the number of different arrangements that can be made with both the D's together and both the S's together. [2]

(iii) Find the number of different arrangements that can be made where the E's are separated by at least one letter and the D's are together. [2]

(iv) Find the number of different arrangements that can be made where the E's are not together, S's are not together and the D's are not together. [4]

6 Emergency flares are simple signalling devices similar to fireworks and they are designed to communicate a much more direct message in an emergency, for example, distress at sea.

A company categorised their stocks of emergency flares as 1-year old, 5-year old and 10-year old. The probabilities of successful firing of 1-year old, 5-year old and 10-year old emergency flares are 0.995, 0.970 and 0.750 respectively.

(i) Find the probability that, out of 100 randomly chosen 1-year old flares, at most 2 fail to fire successfully. [1]

(ii) One-year old flares are packed into boxes of 100 flares. Find the probability that, out of 50 randomly chosen boxes of 1-year old flares, not more than 48 of these boxes will have at most 2 flares that will fail to fire successfully in each box. [3]

(iii) Seven flares are chosen at random, of which one is 5 years old and six are 10 years old. Find the probability that

(a) the 5-year old flare fails to fire successfully and at least 4 of the 10-year old flares fire successfully, [2]

(b) at least 4 of the 7 flares fire successfully. [3]

7 With the move towards automated services at a bank, only two cashiers will be deployed to serve customers wanting to withdraw or deposit cash. For each cashier, the bank observed that the time taken to serve a customer is a random variable having a normal distribution with mean 150 seconds and standard deviation 45 seconds.

(i) Find the probability that the time taken for a randomly chosen customer to be served by a cashier is more than 180 seconds. [1]

(ii) One of the two cashiers serves two customers, one straight after the other. By stating a necessary assumption, find the probability that the total time taken by the cashier is less than 200 seconds. [3]

(iii) During peak-hour on a particular day, one cashier has a queue of 4 customers and the other cashier has a queue of 3 customers, and the cashiers begin to deal with customers at the front of their queues. Assuming that the time taken by each cashier to serve a customer is independent of the other cashier, find the probability that the 4 customers in the first queue will all be served before the 3 customers in the second queue are all served. [3]

8 To study if the urea serum content, u mmol per litre, depends on the age of a person, 10 patients of different ages, x years, admitted into the Accident and Emergency Department of a hospital are taken for study by a medical student. The results are shown in the table below.

Age, x (years)	37	44	56	60	64	71	74	77	81	89
Urea, u (mmol/l)	4.2	5.1	4.9	5.7	7.4	7.0	6.8	6.2	7.8	9.6

(i) Draw a scatter diagram of these data. [1]

(ii) By calculating the relevant product moment correlation coefficients, determine whether the relationship between u and x is modelled better by $u = ax + b$ or by $u = ae^{bx}$. Explain how you decide which model is better, and state the equation in this case. [5]

(iii) Explain why we can use the equation in **(ii)** to estimate the age of the patient when the urea serum is 7 mmol per litre. Find the estimated age of the patient when the urea serum is 7 mmol per litre [2]

(iv) The units for the urea serum is now given in mmol per decilitre.

(a) Give a reason if the product moment correlation coefficient calculated in **(ii)** will be changed. [1]

(b) Given that 1 decilitre is equal to 0.1 litre, re-write your equation in **(ii)** so that it can be used when the urea serum is given in mmol per decilitre. [1]

9 A game is played with 18 cards, each printed with a number from 1 to 6 and each number appears on exactly 3 cards. A player draws 3 cards without replacement. The random variable X is the number of cards with the same number.

(i) Show that $\mathrm{P}(X=2)=\dfrac{45}{136}$ and determine the probability distribution of X. [3]

(ii) Find $\mathrm{E}(X)$ and show that $\mathrm{Var}(X)=0.922$ correct to 3 significant figures. [3]

(iii) 40 games are played. Find the probability that the average number of cards with the same number is more than 1. [2]

(iv) In each game, Sam wins $\$(a+10)$ if there are cards with the same number, otherwise he loses $\$a$. Find the possible values of a, where a is an integer, such that Sam's expected winnings per game is positive. [4]

10 In the manufacturing of a computer device, there is a process which coats a computer part with a material that is supposed to be 100 microns thick. If the coating is too thin, the proper insulation of the computer device will not occur and it will not function reliably. Similarly, if the coating is too thick, the device will not fit properly with other computer components.

The manufacturer has calibrated the machine that applies the coating so that it has an average coating depth of 100 microns with a standard deviation of 10 microns. When calibrated this way, the process is said to be "in control".

Due to wear out of mechanical parts, there is a tendency for the process to drift. Hence the process has to be monitored to make sure that it is in control.

(i) After running the process for a reasonable time, a random sample of 50 computer devices is drawn. The sample mean is found to be 103.4 microns. Test at the 5% level of significance whether the sample suggests that the process is not in control. State any assumptions for this test to be valid. [4]

(ii) To ease the procedure of checking, the supervisor of this process would like to find the range of values of the sample mean of a random sample of size 50 that will suggests that the process is not in control at 5% level of significance. Find the required range of values of the sample mean, leaving your answer to 1 decimal places. [3]

On another occasion, a random sample of 40 computer devices is taken. The data can be summarised by

$$\Sigma(y-100)=164, \quad \Sigma(y-100)^2=9447\,.$$

(iii) Calculate the unbiased estimate for the population mean and population variance of the thickness of a coating on the computer device. [2]

(iv) Give, in context, a reason why we may not be able to use 10 microns for the standard deviation of the thickness of a coating on the computer device. [1]

(v) Assume that the standard deviation has changed, test at the 4% level of significance whether the sample suggests that the process is not in control. [3]

———–END OF PAPER———–