Name:_____________________________________ **Class**_______ **Index No**____

BUKIT PANJANG GOVERNMENT HIGH SCHOOL
PRELIMINARY EXAMINATION 2020
SECONDARY FOUR EXPRESS
SECONDARY FIVE NORMAL ACADEMIC

ADDITIONAL MATHEMATICS	**4047/1**
Paper 1	Date: 21 August 2020
Candidates answer on the question paper.	Time: 0800 – 1000
No additional materials are required.	Duration: 2h

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This paper has a total of 22 printed pages.

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad .$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)......(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \tfrac{1}{2} ab \sin C$$

1 It is given that $\cos A = -m$, where $m > 0$, and that A is obtuse.
Find the value of each of the following in terms of m.

(a) $\tan A$ [2]

(b) $\cot(180 - A)$ [1]

(c) $\cos\left(\frac{A}{2}\right)$ [3]

2 **(i)** Find the range of values of p for which the line $y = 2x + 5$ will meet the curve $y^2 = px$. [4]

(ii) Hence, on the same axes, sketch the graphs of $y = 2x + 5$ and $y^2 = 5x$. [2]

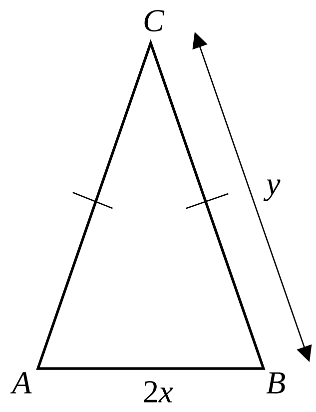
3 The gradient of a curve is $\frac{dy}{dx} = p + \frac{q}{x^3}$, where p and q are constants. The gradient of the normal at $A(1, 4)$ on the curve is -1 and the tangent to the curve at $B(2, 10)$ is $y = 8x - 6$.

(i) Calculate the value of p and of q. [4]

(ii) Using the value of p and of q found in (i), find the equation of the curve. [2]

(iii) Show that gradient increases as x increases. [2]

4 In the diagram below, ΔABC is an isosceles triangle with $BC = y$ cm and $AB = 2x$ cm. It is also given that the perimeter of ΔABC is 50 cm.



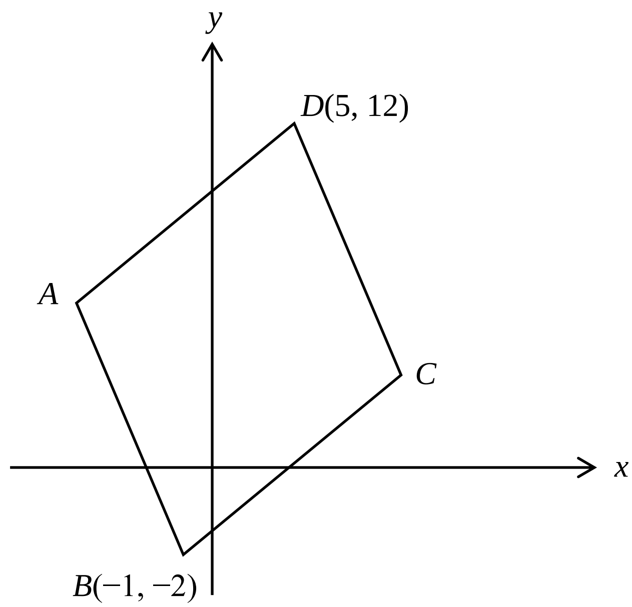
(i) Show that the area of ΔABC is given by $A = x\sqrt{625 - 50x}$. [3]

(ii) Given that x is increasing at a rate of 0.2 cm/s, find the rate at which the area is increasing at the instant when $x = 3$. [3]

5 **(a)** The sum of the coefficients of the first two terms in the expansion, in descending powers of x, of $(1+2x)\left(2x-\frac{1}{x^2}\right)^n$ is 768, where n is a positive integer greater than 2. Show that n is 8. [4]

(b) Find the term containing x^{-7} in the expansion of $\left(2x-\frac{1}{x^2}\right)^8$. [4]

6 **Solutions to this question by accurate drawing will not be accepted.**



In the diagram, *ABCD* is a rhombus. The points *B* and *D* have coordinates $(-1, -2)$ and $(5, 12)$ respectively.

(i) Show that the equation of *AC* is $7y = -3x + 41$. [4]

(ii) Given that a line $5y = 3x + 55$ passes through point *A*, find the coordinates of *A*. [2]

(iii) Find the coordinates of *C*. [2]

(iv) Find the area of the rhombus *ABCD*. [2]

(v) If point *E* lies on *BD* produced such that *BD* : *DE* = 2 : 3, find the coordinates of *E*. [2]

7 **(a)** Given that $3^{n+2} - 3^{n} = \frac{5^{n+1}}{25^{n}}$, find the value of 15^{n}. [3]

(b) Without using a calculator, find the root of the equation $x\sqrt{80} = \sqrt{20} - x\sqrt{48}$ in the form $\dfrac{a + b\sqrt{c}}{4}$. [3]

8 A particle moves in a straight line from a point O such that t seconds after leaving O, its velocity, v m/s, is given by $v = 5(4t-1)^2 - 125$. Find

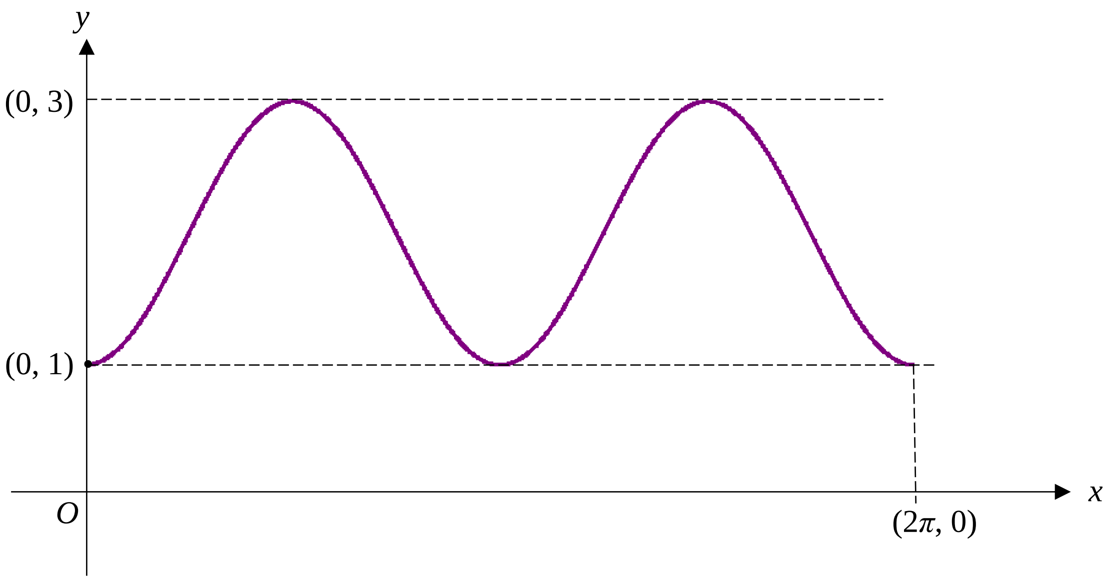
(i) the initial acceleration of the particle, [2]

(ii) the value of t at which the particle is instantaneously at rest, [2]

(iii) the minimum velocity of the particle. [1]

(iv) the distance travelled by the particle in the second second, [5]

9



The diagram shows the curve $y = a + b\cos(cx)$ for $0 \leq x \leq 2\pi$.

(i) Write down the value of a, of b and of c. [3]

(ii) Sketch, on the same diagram, the graph of $y = \sin\left(\frac{x}{2}\right) + 2$ for $0 \le x \le 2\pi$. [3]

(iii) Deduce the largest integer value of k such that $a + b\cos(cx) > \sin\left(\frac{x}{2}\right) + k$ for $0 \le x \le 2\pi$. [1]

10 A curve has the equation $y = x^3 e^{2x}$.

Find the x-coordinates of the stationary points and determine the nature of each point [7]

11 In an experimental environment, the population of a type of insect was observed. Over a period of 10 days from the start of the experiment, the number of insects decreased from 1100 to 600. The insect population is given by the formula $P = A + 900e^{kt}$, where A and k are constants and t is the number of days from the start of the experiment.

(a) Find the value of A and of k. [3]

(b) Explain why the population of the insects approaches the value of A after a long period of time. [1]

End of Paper