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NANYANG JUNIOR COLLEGE

JC2 PRELIMINARY EXAMINATION

Higher 2

Candidate Name	

CT Class	1	8			Centre Number/ Index Number					/					

MATHEMATICS 9758/01

Paper 1

2nd September 2019

3 Hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS

Write your name and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For examiner's use only	
Question number	**Mark**
1	
2	
3	
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8	
9	
10	
11	
12	
Total	

This document consists of **6** printed pages.

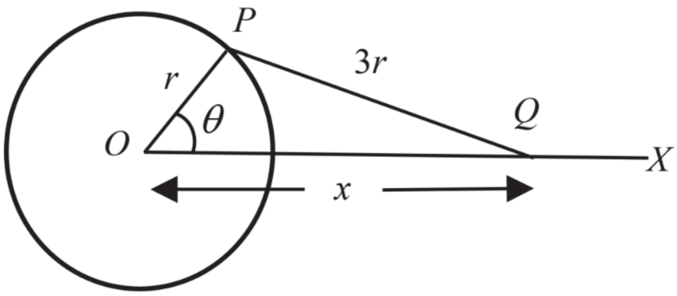
NANYANG JUNIOR COLLEGE
Internal Examinations

[Turn Over

1 The curve with equation $y^2 = x^2 + 9$ is transformed by a stretch with scale factor 2 parallel to the x-axis, followed by a translation of 4 units in the negative x-direction, followed by a translation of $\frac{1}{2}$ units in the positive y-direction.

Find the equation of the new curve and state the equations of any asymptote(s). Sketch the new curve, indicating the coordinates of any turning points. [6]

2 The diagram shows a mechanism for converting rotational motion into linear motion. The point P, on the circumference of a disc of radius r, rotates about a fixed point O. The point Q moves along the line OX, and P and Q are connected by a rod of fixed length $3r$. As the disc rotates, the point Q is made to slide backwards and forwards along OX. At time t, angle POQ is θ, measured anticlockwise from OX, and the distance OQ is x.



(i) Show that $x = r\left(\cos\theta + \sqrt{9 - \sin^2\theta}\right)$. [2]

(ii) State the maximum value of x. [1]

(iii) Express x as a polynomial in θ if θ is sufficiently small for θ^3 and higher powers of θ are to be neglected. [3]

3 Without using a calculator, solve the inequality $\dfrac{x^2 - 3x + 4}{x+2} \geq 2x + 1$. Hence solve the inequality $\dfrac{a^{2x} + 3a^x + 4}{a^x - 2} \leq 2a^x - 1$ where $a > 2$. [6]

4 **(i)** Using double angle formula, prove that $\sin^4\theta = \frac{1}{8}\left(3 - 4\cos 2\theta + \cos 4\theta\right)$. [2]

(ii) By using the substitution $x = 2\cos\theta$, find the exact value of $\displaystyle\int_0^2 \left(4 - x^2\right)^{\frac{3}{2}}\, dx$. [4]

5 Relative to the origin O, the points A, B, and C, have non-zero position vectors $\mathbf{a}$, $\mathbf{b}$, and $3\mathbf{a}$ respectively. D lies on AB such that $AD = \lambda AB$, where $0 < \lambda < 1$.

(i) Write down a vector equation of the line OD. [1]

(ii) The point E is the midpoint of BC. Find the value of λ if E lies on the line OD. Show that the area of ΔBED is given by $k\left|\mathbf{a}\times\mathbf{b}\right|$, where k is a constant to be determined. [5]

6 The function f is given by $\mathrm{f} : x \mapsto 2x^2 + 4x + k$ for $-5 \le x < a$, where a and k are constants and $k > 2$.

(i) State the largest value of a for the inverse of f to exist. [1]

For the value of a found in **(i)**,

(ii) find $\mathrm{f}^{-1}(x)$ and the domain of f^{-1}, leaving your answer in terms of k, [3]

(iii) on the same diagram, sketch the graphs of $y = \mathrm{ff}^{-1}(x)$ and $y = \mathrm{f}^{-1}\mathrm{f}(x)$, labelling your graphs clearly. Determine the number of solutions to $\mathrm{ff}^{-1}(x) = \mathrm{f}^{-1}\mathrm{f}(x)$. [4]

7 A spherical tank with negligible thickness and internal radius a cm contains water. At time t s, the water surface is at a height x cm above the lowest point of the tank and the volume of water in the tank, V cm^3, is given by $V = \frac{1}{3}\pi x^2(3a - x)$. Water flows from the tank, through an outlet at its lowest point, at a rate $\pi k\sqrt{x}$ cm^3 s^{-1}, where k is a positive constant.

(i) Show that $\left(2ax - x^2\right)\frac{\mathrm{d}x}{\mathrm{d}t} = -k\sqrt{x}$. [2]

(ii) Find the general solution for t in terms of x, a and k. [3]

(iii) Find the ratio $T_1 : T_2$, where T_1 is the time taken to empty the tank when initially it is completely full, and T_2 is the time taken to empty the tank when initially it is half full. [4]

8 A curve C has equation $y^2 + xy = 4$, where $y > 0$.

(i) Without using a calculator, find the coordinates of the point on C at which the gradient is $-\frac{1}{5}$. [4]

(ii) Variables z and y are related by the equation $y^2 + z^2 = 10y$, where $z > 0$. Given that x increases at a constant rate of 0.5 unit/s, find the rate of change of z when $x = 3$. [5]

9 **(a)** The complex numbers z and w satisfy the simultaneous equations

$$|z| - w^* = -3 - \sqrt{2}\mathrm{i} \text{ and } w^* + w + 5z = 1 + 20\mathrm{i},$$

where w^* is the complex conjugate of w. Find the value of z and the corresponding value of w. [4]

(b) It is given that 8i is a root of the equation $\mathrm{i}z^3 + (8 - 2\mathrm{i})z^2 + az + 40 = 0$ where a is a complex number.

(i) Find a. [2]

(ii) Hence, find the other roots of the equation, leaving your answer in the form $a + b\mathrm{i}$ where a and b are real constants. [3]

(iii) Deduce the number of real roots the equation $z^3 - (8 - 2\mathrm{i})z^2 + a\mathrm{i}z + 40 = 0$ has. [1]

10 For this question, you may use the results $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$ and $\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$.

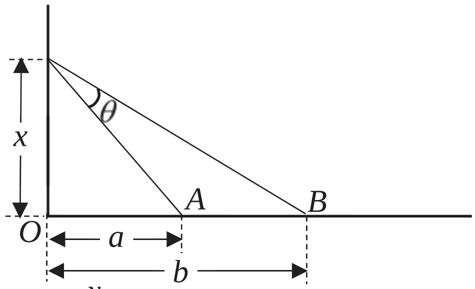
(i) Find $\sum_{r=1}^{n} r^2(2r-1)$ in terms of n. [2]

(ii) Find $\sum_{r=1}^{n} r^2(r-1)$ in terms of n. Hence find $\sum_{r=2}^{n-1} r(r+1)^2$ in terms of n. [5]

(iii) Without using a graphing calculator, find the sum of the series

$$4(25) - 5(36) + 6(49) - 7(64) + \cdots - 59(3600).$$ [3]

11 A boy is playing a ball game on a field. He arranges two cones A and B along the end of the field such that the cones are a and b metres respectively from one corner, O, of the field as shown in the diagram below. The boy stands along the edge of the field at x metres from O and kicks the ball between the two cones. The angle that the two cones subtends at the position of the boy is denoted by θ.



(i) Show that $\tan\theta = (b-a)\dfrac{x}{x^2+ab}$. [2]

(ii) It is given that $a = 15$ and $b = 20$. Find by differentiation, the value of x such that θ is at a maximum. [3]

(iii) It is given instead that the boy gets two friends to vary the position of both cones A and B along the end of the field such that $5 \le a \le 12$ and $b = 2a$, and the boy moves along the edge of the field such that his distance from cone A remains unchanged at 18 metres. Sketch a graph that shows how θ varies with a and find the largest possible value of θ . [4]

(iv) The boy runs until he is at a distance k metres from the goal line that is formed by the two cones and kicks the ball toward the goal line. The path of the ball is modelled by the equation $h = -\left(\dfrac{1}{10}k + 2\right)^2 + 6$, where k is the distance of the ball from the goal line and h its corresponding height above the ground respectively. Find the angle that the path of the ball makes with the horizontal at the instant the ball crosses the goal line. [3]

12 In the study of force field, we are often interested in whether the work done in moving an object from point A to point B is independent of the path taken. If a force field is such that the work done is independent of the path taken, it is said to be a *conservative* field.

A force field **F** can be regarded as a vector $\mathbf{F} = M(x, y)\mathbf{i} + N(x, y)\mathbf{j}$ where M and N are functions of x and y. The path that the object is moving along is denoted by C. The work done in moving the object along the curve C from the point where $x = a$ to the point where $x = b$ is given by

$$W = \int_a^b \left[M(x, y) + N(x, y)\frac{\mathrm{d}y}{\mathrm{d}x} \right] \mathrm{d}x,$$

where $y = \mathrm{f}(x)$ is the equation of the curve C.

(i) Sketch the curve C with equation $y^2 = 4(1 - x)$, for $x \leq 1$. [2]

(ii) Find an expression of $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of y. [1]

(iii) The points P and Q are on C with $x = 1$ and $x = -3$ respectively and Q is below the x-axis. Find the equation of the line PQ. [2]

For the rest of the question, the force field is given by $\mathbf{F} = x^2\mathbf{i} + xy^2\mathbf{j}$.

(iv) Show that the work done in moving an object along the curve C from Q to P is given by the integral $\int_{-3}^{1} \left(x^2 + 4x(1 - x)^{\frac{1}{2}} \right) \mathrm{d}x$. Hence evaluate the exact work done in moving the object along the curve C from Q to P. [4]

(v) Find the work done in moving an object along the line PQ from Q to P to 2 decimal places. [2]

(vi) Determine, with reason, whether **F** is a conservative force field. [1]

–––––END OF PAPER–––––