JURONG PIONEER JUNIOR COLLEGE

JC2 Preliminary Examination 2019

MATHEMATICS **9758/01**

Higher 2 **18 September 2019**

Paper 1 **3 hours**

1 **(i)** The first four terms of a sequence are given by $u_1 = -13$, $u_2 = -12.8$, $u_3 = 1.8$ and $u_4 = 38$. Given that u_m is a cubic polynomial in *m*, find u_m in terms of *m*. [3]

(ii) Find the range of values of *m* for which u_m is greater than 2000. [2]

2 **(a)** Express $y = \dfrac{3x-1}{x-2}$ in the form $y = A + \dfrac{B}{x-2}$, where *A* and *B* are constants to be found. Hence, state a sequence of transformations that transforms the graph of $y = \dfrac{1}{x}$ to the graph of $y = \dfrac{3x-1}{x-2}$. [4]

(b) It is given that $\mathrm{g}(x) = x^2 - 2x + 2$. Sketch the graph of $y = \mathrm{g}(|x|)$, stating clearly the coordinates of any turning points and axial intercepts. Find numerically, the volume of revolution when the region bounded by the curve $y = \mathrm{g}(|x|)$ and the line $y = 5$ is rotated completely about the *x*-axis. [5]

3 Referred to an origin *O*, the points *A* and *B* are such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. The point *C* is such that *OACB* is a parallelogram. The point *D* is on *BC* such that $\overrightarrow{BD} = \lambda\overrightarrow{BC}$ and the point *E* is on *AC* such that $\overrightarrow{AE} = \mu\overrightarrow{AC}$, where λ and μ are positive constants. The area of triangle *ODE* is *k* times the area of triangle *OCE*.

(i) By finding the area of triangle *ODE* and *OCE* in terms of **a** and **b**, find *k* in terms of μ and λ. [6]

(ii) The point *F* is on *OC* and *ED* such that $OF : FC = 6 : 1$ and $DF : FE = 3 : 4$. By finding the values of λ and μ, calculate the value of *k*. [3]

4 The curve *C* has equation $y = \dfrac{x^2+5}{x-2}$.

(i) Prove, using an algebraic method, that *C* cannot lie between two values to be determined. [4]

(ii) Sketch *C*, showing clearly the equations of any asymptotes and coordinates of any turning points and axial intercepts. [4]

(iii) By adding a suitable graph to your sketch in **(ii)**, deduce the range of values of *h* for which the equation

$$\left(x^2+5\right)^2 + (x+1)^2(x-2)^2 = h^2(x-2)^2$$

has at least one positive real root. [3]

[Turn over

5 **Do not use a graphing calculator in answering this question.**

(a) **(i)** It is given that $w_1 = -3+\sqrt{5}\,\mathrm{i}$. Find the value of w_1^3, showing clearly how you obtain your answer. [2]

(ii) Given that $-3+\sqrt{5}\,\mathrm{i}$ is a root of the equation

$$4w^3 + pw^2 + qw - 14 = 0,$$

using your result in **(i)** , find the values of the real numbers p and q. [3]

(iii) For these values of p and q, find the other two roots of the equation in part **(ii)**. [3]

(b) It is given that $z = -1-\sqrt{3}\,\mathrm{i}$.

Find the set of values of n for which $\dfrac{z^*}{z^n}$ is purely imaginary. [4]

6 The function f is defined for all real x by

$$\mathrm{f}(x) = \mathrm{e}^{2x} - 9\mathrm{e}^{-2x}.$$

(i) Show that $\mathrm{f}'(x) > 0$ for all x. [2]

(ii) Show that the set of values of x for which the graph $y = \mathrm{f}(x)$ is concave upward is the same as the set of values of x for which $\mathrm{f}(x) > 0$, and find this set of values of x, in the form of $k\ln 3$, where k is a constant to be found. [3]

(iii) Sketch the graph of $y = \mathrm{f}(x)$, showing clearly any points of intersections with the axes. [2]

(iv) Hence, find the exact value of $\int_0^2 \left|\mathrm{e}^{2x} - 9\mathrm{e}^{-2x}\right| \mathrm{d}x$. [4]

7 It is given that

$$\mathrm{f}(x) = \begin{cases} 4a^2 - x^2, & \text{for } 0 < x \le 2a, \\ 2a(x-2a), & \text{for } 2a < x \le 4a, \end{cases}$$

and that $\mathrm{f}(x) = \mathrm{f}(x+4a)$ for all real values of x, where a is a positive real constant.

(i) Evaluate $\mathrm{f}(2019a)$ in terms of a. [1]

(ii) Sketch the graph of $y = \mathrm{f}(x)$ for $-3a \le x \le 5a$. [3]

The function g is defined by

$$\mathrm{g} : x \mapsto \sqrt{4a^2 - (x-2a)^2}, \qquad 2a < x < 4a.$$

(iii) Determine whether the composite function gf exists, justifying your answer. [1]

(iv) Give, in terms of a, a definition of fg. [2]

(v) Given that $(\mathrm{fg})^{-1}(27) = \dfrac{7}{2}a$, find the exact value of a. [2]

8

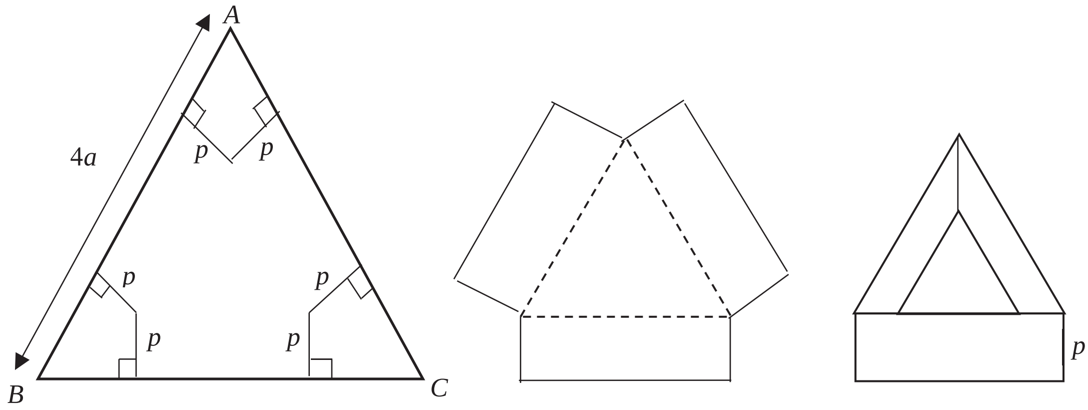

Fig.1 **Fig.2** **Fig.3**

Fig. 1 shows a metal sheet, ABC, in the form of an equilateral triangle of side $4a$ cm. A kite shape is cut from each corner, to give the shape as shown in Fig. 2. The remaining metal sheet shown in Fig. 2 is bent along the dotted lines, to form an open triangular prism of height p cm shown in Fig. 3.

(i) Show that the volume of the prism is given by $V = \sqrt{3}p\left(2a - \sqrt{3}p\right)^2 \text{ cm}^3$. [3]

(ii) Without using a calculator, find in terms of a, the exact value of p that gives a stationary value of V, and explain why there is only one answer. [6]

(iii) The prism is used by a housewife as a mould for making a dessert. To make the dessert, The housewife has to fill up $\frac{3}{4}$ of the mould with coconut milk. The cost of coconut milk is 0.4 cents per cm^3. What is the exact maximum cost in terms of a she needs to pay for the coconut milk? [3]

9 Find

(a) $\int \frac{e^{\frac{1}{x}}}{x^2}\,dx$, [2]

(b) $\int \cos kx \cos(k+2)x\,dx$, where k is a positive constant, [2]

(c) $\int x \tan^{-1}(3x)\,dx$. [6]

10 **(a)** The Deep Space spacecraft launched in October 1998 used an ion engine to travel from Earth to the Comet Borrelly. The average speed of the spacecraft in October 1998 was 44 000 km/hr. The monthly average speed, v_n of the spacecraft in month n based on its first 5 months of operation was given by:

Month, n	1	2	3	4	5
Average speed, v_n	44 000	44335	44 670	45 005	45 340

Assume that v_n follows the same increment for the rest of its flight.

(i) State a general formula for v_n in terms of n. [1]

(ii) In which month and year did the average speed of the spacecraft first exceed 53 500 km/hr ? [3]

[Turn over

(iii) Assume that there are 30 days per month. It is known that the total distance travelled by the spacecraft from Earth is given by $\sum_{r=1}^{n}(v_r T)$ where T is the time taken, in hours, by the spacecraft to travel in one month. Given that the spacecraft travelled from Earth continuously for 3 years to reach Comet Borrelly, find the total distance that it travelled. [3]

(b) Dermontt's Law is an empirical formula for the orbital period of major satellites orbiting planets in the solar system. It is represented by the equation $T_n = T_0 C^n$, where T_n is the orbital period, in days, of the $(n + 1)^{\text{th}}$ satellite and C is a constant associated with the satellite system in question. It is known that the planet Jupiter has 67 satellites. The orbital period of its first satellite is 0.44 days and $C = 2.03$.

(i) Find the longest orbital period of a satellite of Jupiter. [2]

(ii) Find the largest value of n for which the total orbital periods of the first n satellites of Jupiter is within 5×10^6 days of the orbital period of the 20^{th} satellite of Jupiter. [3]