Name:_________________________________ Class_______ Index No____

BUKIT PANJANG GOVERNMENT HIGH SCHOOL
PRELIMINARY EXAMINATION 2020
SECONDARY FOUR EXPRESS
SECONDARY FIVE NORMAL ACADEMIC

ADDITIONAL MATHEMATICS	**4047/2**
Paper 2	Date: 26 August 2020
Candidates answer on the question paper.	Time: 08 00 – 10 30
Additional materials: Graph paper	Duration: 2h 30 min

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This paper has a total of 21 pages.

Setter: Mrs Chiu H W

[Turn over]

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad .$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \tfrac{1}{2} ab \sin C$$

1 The expression $f(x) = x^3 + ax^2 + bx + c$ leaves the same remainder, R, when it is divided by $x + 2$ and when it is divided by $x - 2$.

(i) Evaluate b. [2]

$f(x)$ also leaves the same remainder, R, when divided by $x - 1$.

(ii) Evaluate a. [2]

$f(x)$ leaves a remainder of 4 when divided by $x - 3$.

(iii) Evaluate c. [1]

2 The equation of a polynomial is given by p(x) = $2x^3 - x^2 + 16x - 8$.

(i) Show that $2x - 1$ is a factor of p(x). [1]

(ii) Show that p(x) = 0 has only one real root. [3]

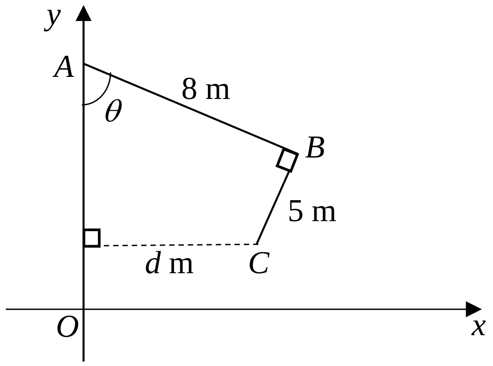
(iii) Express $\dfrac{x^2+2x+40}{2x^3-x^2+16x-8}$ in partial fractions. [5]

(iv) Hence find $\displaystyle\int \frac{x^2+2x+40}{2x^3-x^2+16x-8}\,dx$ [3]

3 (a) Solve the equation $\log_2(x+2) - \log_{\sqrt{2}}(x-1) = 3$. [5]

(b) The curve $y = ax^b + 7$, where a and b are constants, passes through the points $(2, 47)$, $(-3, -128)$ and $(5, k)$. Find the values of a, b and k. [5]

4 The diagram shows a rod AB which is hinged at A, and a rod BC which is fixed at B such that angle $ABC = 90°$. The rods can move in the xy-plane with origin O where the x and y axes are horizontal and vertical respectively. The rod AB can turn about A and is inclined at an angle θ to the y-axis, where $0° \leq \theta \leq 180°$. The lengths of AB and BC are 8 m and 5 m respectively.



Given that C is d m from the y-axis,

(i) find the values of a and b for which $d = a \sin \theta - b \cos \theta$ [2]

Using the values of a and b found in part (i),

(ii) express d in the form $R \sin (\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

Hence

(iii) explain if it is possible for d to be 10 m, [2]

(iv) find the value(s) of θ when $d = 6$ m. [2]

5 (a) It is given that $f(x) = \ln\sqrt[3]{\dfrac{5+x}{5-x}}$.

(i) Find $f'(x)$ and $f''(x)$. [4]

(ii) Hence determine the range of values of x for which both $f'(x)$ and $f''(x)$ are positive. [4]

(b) Show that $\frac{d}{dx}\left[4\sin^2\left(\frac{x}{2}+\pi\right)\right]=k\sin x$ where k is a constant. [3]

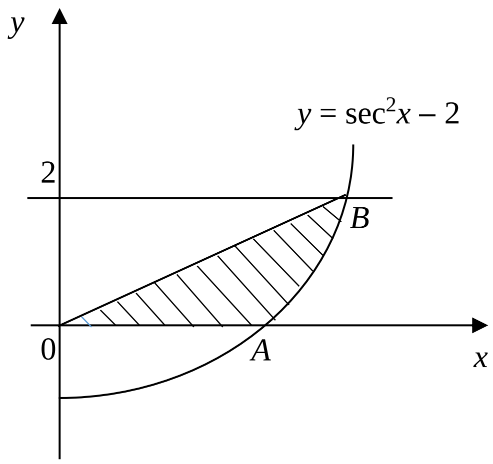
6 (i) Prove that $\dfrac{\operatorname{cosec}^2\theta - 2}{\operatorname{cosec}^2\theta} = \cos 2\theta$. [3]

(ii) Hence solve the equation $\dfrac{\operatorname{cosec}^2\theta - 2}{\operatorname{cosec}^2\theta} + \dfrac{3}{\operatorname{cosec} 2\theta} = 0$ for $0 < \theta < 5$. [4]

7 A quadratic equation with integer coefficients has roots α and β.

Given that $\alpha - \beta = 2$ and $\alpha^2 - \beta^2 = 3$, find the quadratic equation **without** calculating the values of α and β. [5]

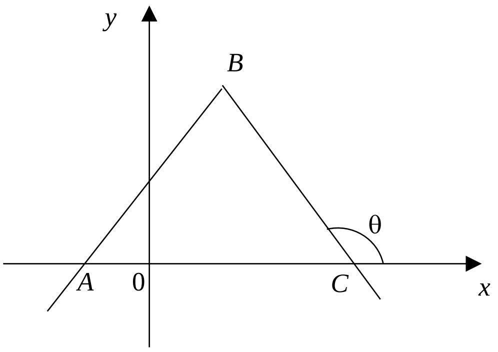
8 The diagram shows the line $y = 2$ and part of the curve $y = \sec^2 x - 2$. The curve intersects the x-axis at the point A and line $y = 2$ at the point B. A straight line through the origin intersects the curve at point B.



(i) Find the x-coordinate of A and B. Express your answers in terms of π. [3]

(ii) Determine the area of the shaded region bounded by the curve, the x-axis and the line OB. Give your answer as exact value. [5]

9



The diagram shows part of the graph of $y = 3 - |1 - 2x|$.

(i) Find the coordinates of the points *A*, *B* and *C*. [3]

(ii) The line *BC* makes an angle θ with the *x*-axis. Find the value of tan θ. [1]

(iii) Solve the equation $3-|1-2x|=2x-x^2$. [4]

(iv) Without solving for *x*, explain why there is no real solution for the equation stated below.

$$3-|1-2x|=4+x^2$$ [3]

10 A circle, C_1, and another circle, C_2, pass through the same point (0 , –3).

(i) Given that the radius of both circles is $\sqrt{5}$ units and their centres lie on the line $y = x$, find the equations of C_1 and C_2. [5]

(ii) Circle, C_1 and circle, C_2, intersect at a point on the *x*-axis. Find the *x*-coordinate of the point of intersection of C_1 and C_2 on the *x*-axis . [3]

(iii) Given that a point *P* lies on circle, C_1 and another point *Q* lies on circle, C_2, find the greatest distance between *P* and *Q*. [3]

11 The table below shows experimental values of two variables, x and y.

x	1	2	3	4	5
y	0.50	2.12	3.18	4.00	4.70

It is known that x and y are related by the equation $y = \frac{a}{\sqrt{x}} + b\sqrt{x}$ where a and b are constants.

(i) Plot $\frac{y}{\sqrt{x}}$ against $\frac{1}{x}$ and draw a straight line. [3]

(ii) Use your graph to estimate the value of each of the constants a and b. [3]

(iii) By drawing another straight line on the graph in part (i), solve the following simultaneous equations. [5]

$$y = \frac{a}{\sqrt{x}} + b\sqrt{x}$$

$$y\sqrt{x} = 3$$

– END –

1 (i) $b = -4$

(ii) $a = -1$

(iii) $c = -2$

2 (i) Show $p\left(\frac{1}{2}\right) = 0$

(ii) $x = \frac{1}{2}$

$\left(x^2 + 8\right) = 0$ no real solution

(iii) $\frac{5}{2x-1} - \frac{2x}{x^2+8}$

(iv) $\frac{5}{2}\ln(2x-1) - \ln\left(x^2+8\right) + c$

3 (a) $x = 1.68$, 0.447 (rejected)

(b) $a = 5$, $b = 3$, $k = 632$

4 (i) $d = 8\sin\theta - 5\cos\theta$

(ii) $d = \sqrt{89}\ \sin(\theta - 32.0°)$

(iii) Max value of $d = \sqrt{89} = 9.43$

when $\sin(\theta - 32.0°) = 1$. So not possible for d to be 10 m.

(iv) 71.5° , 172.5°

5 (a) (i) $f'(x) = \frac{10}{3\left(25-x^2\right)}$

$f''(x) = \frac{20x}{3\left(25-x^2\right)^2}$

(ii) $0 < x < 5$

(b) $\frac{d}{dx}\left[4\sin^2\left(\frac{x}{2}+\pi\right)\right] = 2\sin x$

6 (ii) 1.41 , 2.98 , 4.55

7 $16x^2 - 24x - 7 = 0$

8 (i) x-coordinate of $A = \frac{\pi}{4}$

x-coordinate of $B = \frac{\pi}{3}$

(ii) $\left(1 - \sqrt{3} + \frac{\pi}{2}\right)$ units^2

9 (i) $A(-1, 0)$, $B\left(\frac{1}{2}, 3\right)$

$C(2, 0)$

(ii) $\tan\theta = -2$

(iii) $x = 2$

(iv) Max value of $3 - |1-2x|$ is 3 when $x = \frac{1}{2}$.

Min value of $4 + x^2$ is 4 when $x = 0$.

The curve and line do not intersect so there is no real solution.

10 (i) $C_1 : (x+1)^2 + (y+1)^2 = 5$

$C_2 : (x+2)^2 + (y+2)^2 = 5$

(ii) $x = -3$

(iii) $\sqrt{2} + 2\sqrt{5}$ or 5.89 units

11 (ii) $a = -2$, $b = 2.5$

(iii) Draw the line $\frac{y}{\sqrt{x}} = \frac{3}{x}$.

Point of intersection is

(0.5 , 1.5)

$x = 2$, $y = 2.12$