Name:		Index Number:		Class:	

DUNMAN HIGH SCHOOL
Preliminary Examination
Year 6

MATHEMATICS (Higher 2) 9758/02

Paper 2

September 2019

3 hours

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For teachers' use:

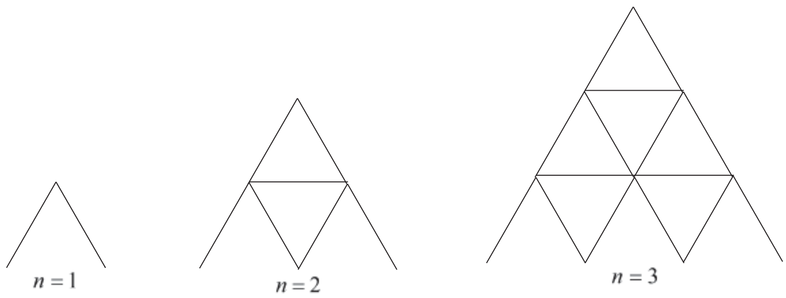
Qn	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Total
Score												
Max Score	**7**	**8**	**8**	**8**	**9**	**4**	**7**	**9**	**12**	**12**	**16**	**100**

 This question paper consists of **23** printed pages and **1** blank page.

Section A: Pure Mathematics [40 marks]

Commented [jo1]: APY

1 Ryan has playing cards which he stacks into pyramids. He will begin by stacking up cards to form a pyramid with 1 level, followed by another pyramid with 2 levels and so forth. The pyramids with n levels for different values of n are shown below:



Let S_n denotes the number of cards in a pyramid with n levels. It is given that $S_n = an^2 + bn + c$ for some constants a, b and c.

(i) Give an expression of the number of additional cards needed to form a pyramid of nth level from $(n-1)$th level. Leave your expression in terms of a, b and n. [2]

(ii) Find the values of a, b and c. [2]

(iii) Hence prove that S_n is the sum of an arithmetic progression and state the common difference. [2]

(iv) One pyramid of each level from 1 to 23 is formed. Find the total number of cards required to form these 23 pyramids. [1]

2 A curve C has equation $3x^2 - 2xy + 5y^2 = 14$.

Commented [jo2]: CHC – new qn

(i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x - y}{x - 5y}$. [2]

(ii) Find the exact x-coordinates of the points on the curve C at which the tangent is parallel to the y-axis. [3]

(iii) A point $P(x, y)$ moves along the curve C in such a way that y decreases at a constant rate of 7 units per second. Given that x increases at the instant when $y = 1$, find the corresponding rate of change in x. [3]

3 The complex number z is such that $az^2 + bz + a = 0$ where a and b are real constants. It is given that $z = z_0$ is a solution to this equation where $\text{Im}(z_0) \neq 0$.

Commented [OMFJ3]: HG - ok

(i) Verify that $z = \dfrac{1}{z_0}$ is the other solution. Hence show that $|z_0| = 1$. [4]

Take $\text{Im}(z_0) = \frac{1}{2}$ for the rest of the question.

(ii) Find the possible complex numbers for z_0. [2]

(iii) If $\text{Re}(z_0) > 0$, find *b* in terms *a*. [2]

4 The complex number w has modulus $\sqrt{2}$ and argument $\frac{1}{4}\pi$ and the complex number z has modulus $\sqrt{2}$ and argument $\frac{5}{6}\pi$.

Commented [jo4]: APY

(i) By first expressing w and z in the form $x+\mathrm{i}y$, find the exact real and imaginary parts of $w+z$. [3]

(ii) On the same Argand diagram, sketch the points P, Q, R representing the complex numbers z, w and $z+w$ respectively. State the geometrical shape of the quadrilateral $OPRQ$. [3]

(iii) Referring to the Argand diagram in part **(ii)**, find $\arg(w+z)$ and show that $\tan\left(\frac{11}{24}\pi\right)=\dfrac{a+\sqrt{2}}{\sqrt{6}+b}$ where a and b are constants to be determined. [2]

5 The curves C_1 and C_2 have equations $y = \dfrac{x-b}{x-a}$ and $y = \dfrac{x-b}{b}$ respectively, where a and b are constants with $1 < a < b$.

Commented [OMFJ5]: JO

(i) Show that the x-coordinates of the points of intersection of C_1 and C_2 are b and $a+b$. Hence sketch C_1 and C_2 on a single diagram, labelling any points of intersection with the axes and the equations of any asymptotes. [4]

(ii) Using the diagram, solve $\dfrac{x-b}{x-a} \geq \dfrac{x-b}{b}$. [2]

(iii) Let $a = 2$ and $b = 3$. The region bounded by C_1 and C_2 is rotated through 4 right angles about the y-axis to form a solid of revolution of volume V. Find the numerical value of V, giving your answer correct to 3 decimal places. [3]

Commented [jo6]: new qn to add in volume

Section B: Probability and Statistics [60 marks]

6 Nine gifts, three of which are identical and the rest are distinct, are distributed among five people without restrictions on the number of gifts a person can have. By first considering the number of ways to distribute the distinct gifts or otherwise, find the number of way that the nine gifts can be distributed. [4]

Commented [jo7]: check answer

7 In a school survey, a group of 80 students are asked about how much time per week (to nearest hour) they spend on their co-curricular activities (CCA). The readings are shown below:

Commented [jo8]: EC - ok

	CCA (hours)		
	3 or less	**4 to 6**	**7 or more**
Boy	17	20	10
Girl	18	$15-k$	k

A student is selected random from the group. Defining the events as follows:

G : The student is a girl.
L : The student spends 6 hours or less weekly.
M : The student spends 4 hours or more weekly.

Find the following probabilities in terms of k.

(i) $\mathrm{P}(L' \cup M')$ [2]

(ii) $\mathrm{P}(G \mid L')$ [1]

(iii) Given that $\mathrm{P}(L \cap M) = \frac{2}{5}$, find the value of k. Hence determine if L and M are independent, justifying your answer. [3]

(iv) If the events G and $(L \cap M)$ are mutually exclusive, find the value of k. [1]

8 Sharron who is an amateur swimmer has been attending swimming lessons. She records her time taken to swim 50 metres each month. Her best timing, t seconds, recorded each month x, for the first 7 months is as follows.

Month x	1	2	3	4	5	6	7
Time taken, t	115	87	75	67	62	61	55

(i) Draw a scatter diagram showing these timings. [1]

(ii) It is desired to predict Sharron's timings on future swims. Explain why, in this context, neither a linear nor a quadratic model is likely to be appropriate. [2]

Commented [jo9]: Nat – ok

It is decided to fit a model of the form $t = a + \frac{b}{x}$ where a and b are constants.

(iii) State with a reason whether each of a and b is positive or negative. [2]

(iv) Find the product moment correlation coefficient and the constants a and b. [2]

At the 8th month, Sharron recorded her best timing and calculated the regression line using all the data from the first 8 months to be $t = 48.28 + \frac{69.45}{x}$.

(v) Find her best timing, to the nearest second, at the 8th month. [2]

9 The time taken, T (in minutes), for a 17-year-old student to complete a 5-km run is a random variable with mean 30. After a new training programme is introduced for these students, a random sample of n students is taken. The mean time and standard deviation for the sample are found to be 28.9 minutes and 4.0 minutes respectively.

Commented [OMFJ10]: YCH – shared H1 (use as contextual)

(a) Find the unbiased estimate of the population variance in terms of n. [1]

(b) Using $n = 40$,

(i) carry out a test at the 10% significance level to determine if the mean time taken has changed. State appropriate hypotheses for the test and define any symbols you use. [4]

(ii) State what it means by the p-value in this context. [1]

(iii) Give a reason why no assumptions about the population are needed in order for the test to be valid. [1]

(c) The trainer claims instead that the new training programme is able to improve the mean of T, 30 minutes, by at least 5%. The school wants to test his claim.

(i) Write down the null and alternative hypothesis. [1]

(ii) Using the existing sample, the school carried out a test at 1% significance level and found that there was sufficient evidence to reject the trainer's claim. Find the set of values that n can take, stating any necessary assumption(s) needed to carry out the test. [4]

Commented [jo11]: FM marker in case of t-test

Commented [jo12]: check if want to mark down

10 The speeds of an e-scooter (X km/h) and a pedestrian (Y km/h) measured on a particular stretch of footpath are normally distributed with mean and variance as follows:

	mean	variance
X	12.3	9.9
Y	μ	σ^2

It is known that $\mathrm{P}(Y < 5.2) = \mathrm{P}(Y \geq 7.0) = 0.379$.

(i) State the value of μ and find the value of σ. [2]

(ii) Given that the speeds of half of the e-scooters measured are found to be within a km/h of the mean, find a. [2]

Commented [jo13]: EC – OK (use as contextual)

(iii) A LTA officer stationed himself at the footpath and measured the speeds of 50 e-scooters at random. Find the probability that the 50th e-scooter is the 35th to exceed LTA's legal speed limit of 10 km/h. [3]

(iv) On another day, the LTA officer randomly measured the speeds of 6 e-scooters and 15 pedestrians. Find the probability that the mean speed of the e-scooters is more than twice the mean speed of the pedestrians captured. [3]

(v) Find the probability that the mean speed of n randomly chosen e-scooters is more than 10 km/h, if n is large. [2]

11 **(a)** At a funfair, Alice pays \$3 to play a game by tossing a fair dice until she gets a '6'. Let X be the number of times that the player tosses a fair dice until he gets a '6'. The prize, S (in dollars), that the player may win is given by the following function:

$$S = \begin{cases} 8, & \text{if } X = 1, \\ 4, & \text{if } 2 \le X \le k, \\ 0, & \text{otherwise.} \end{cases}$$

where k is a positive integer.

(i) Show that $\mathrm{P}(2 \le X \le k) = \left(\frac{5}{6}\right) - \left(\frac{5}{6}\right)^k$. Hence draw up a table showing the probability distributions of S. [4]

Commented [jo14]: CHC – ok (to update alternative solution)

(ii) Find the least value of k such that Alice is expected to earn a profit. [3]

(b) Alice uses a computer program to simulate 80 tosses of a biased coin. Let Y be the random variable denoting the number of heads obtained and p be the probability of obtaining a head. It is given that $80+\mathrm{E}(Y)=6\mathrm{Var}(Y)$.

(i) Find the exact value of p. [3]

(ii) Find the probability of obtaining at least 30 heads, given that the first 5 tosses are heads. [3]

(iii) Alice executes the program 50 times. Find the probability that the mean number of heads, $\overline{Y}$, is less than 25. [3]