Name:		Centre/Index Number:		Class:	

DUNMAN HIGH SCHOOL
Preliminary Examination
Year 6

MATHEMATICS (Higher 2) 9758/01

Paper 1

14 September 2022

3 hours

Candidates answer on the Question Paper

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your Name, Index Number and Class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

For teachers' use:

Qn	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Total
Score												
Max Score	**7**	**6**	**7**	**6**	**8**	**9**	**9**	**12**	**12**	**12**	**12**	**100**

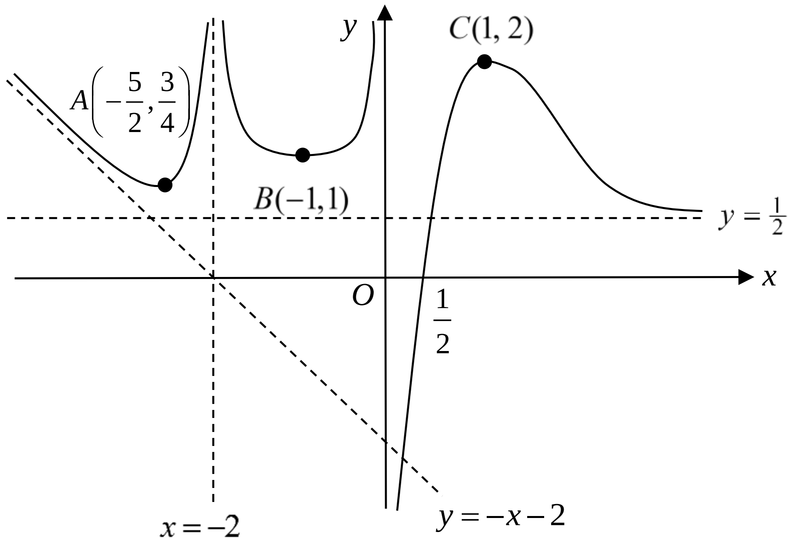
 This consists of __ printed pages (including this cover page).

1 **(a)** Show that $-x^2+6x-14$ is always negative. [1]

(b) Without using a calculator, solve the inequality $\dfrac{x-5}{x-2} \geq \dfrac{3}{4-x}$, giving your answer in exact form. [3]

(c) Hence find the solution for the inequality $\dfrac{\ln x+5}{\ln x+2} \geq \dfrac{3}{4+\ln x}$. [3]

2 The diagram below shows the graph of $y=\mathrm{f}(x)$. The curve crosses the x-axis at $x=\frac{1}{2}$ and has turning points at $A\left(-\frac{5}{2},\frac{3}{4}\right)$, $B(-1,1)$ and $C(1,2)$. It has asymptotes $y=-x-2$, $y=\frac{1}{2}$, $x=0$ and $x=-2$.



On separate diagrams, sketch the following graphs. In each case, show clearly the equations of any asymptotes, coordinates of any points of intersection with both axes and the points corresponding to A, B and C.

(a) $y=\dfrac{1}{\mathrm{f}(x)}$ [3]

(b) $y=\mathrm{f}'(x)$ [3]

3 **(a)** Differentiate $\mathrm{e}^{\sin^2 2x}$ with respect to x. [2]

(b) Find $\displaystyle\int \frac{\mathrm{e}^{\sin^2 2x}\sin 4x}{\sqrt{1+\mathrm{e}^{\sin^2 2x}}}\,\mathrm{d}x$. [2]

(c) Find the exact value of $\displaystyle\int_0^{\frac{\pi}{4}} \mathrm{e}^{\sin^2 2x}\sin 4x\cos^2 2x\,\mathrm{d}x$. [3]

4 For this question, you may use the result $\left(\cos\theta + \mathrm{i}\sin\theta\right)^n = (\cos n\theta + \mathrm{i}\sin n\theta)$.

Find the exact value(s) of θ, where $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, given that

$$(\cos\theta + \mathrm{i}\sin\theta)^3 + (\cos\theta + \mathrm{i}\sin\theta)^5 = -\frac{1}{2} - \frac{\sqrt{3}}{2}\mathrm{i}\,.$$ [6]

5 A curve C has equation $y^2 = 2\sin x + 2xy$, where $0 \le x \le 2\pi$.

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\cos x + y}{y - x}$. [2]

(b) Hence find the coordinates of the stationary points and use the second derivative test to determine its nature. [6]

6 Let $y = \cos\left[\ln(1+x)\right]$ for $|x| < 1$.

(a) Show that $(1+x)^2 \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + (1+x)\frac{\mathrm{d}y}{\mathrm{d}x} + y = 0$. By further differentiation of this result, show that the Maclaurin series for y is given by

$$1 - \tfrac{1}{2}x^2 + \tfrac{1}{2}x^3 - \tfrac{5}{12}x^4 + \ldots.$$ [6]

(b) By using the result in part **(a)**, deduce the Maclaurin series for $\sin\left[\ln(1+x)\right]$ in ascending powers of x up to and including x^3. [2]

(c) Hence state the tangent to the curve $y = \sin\left[\ln(1+x)\right]$ at $x = 0$. [1]

7 **(a)** **(i)** Find, in terms of n and x, an expression for $\sum_{r=0}^{n} \frac{\left(x+3\right)^r}{4^{r+1}}$. [2]

(ii) Give a reason why the infinite series $\sum_{r=0}^{\infty} \frac{\left(x+3\right)^r}{4^{r+1}}$ converges when $x = -5$ and determine its value. [2]

(b) **(i)** Given that $\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$, find in terms of k, the sum $\sum_{r=6}^{2k} r\left(3r-2\right)$. [3]

(ii) Using your result in part **(b)(i)**, evaluate $\sum_{r=10}^{66} \left(r-4\right)\left(3r-14\right)$. [2]

8 The function f is defined as follows:

$$\mathrm{f}: x \mapsto \ln x^2 + 2x + 1 \quad \text{for } x \in \mathbb{R}, x \neq 0.$$

Another function g, defined for $x \in \mathbb{R}$, has the following properties:

- The function g is a continuous decreasing function with $\mathrm{g}(0) = -1$.
- The graph of $y = \mathrm{g}(x)$ has exactly one asymptote. The equation of this asymptote is $y = 0$.

(a) Sketch, on separate diagrams, the graph of $y = \mathrm{f}(x)$ and a possible graph for $y = \mathrm{g}(x)$. [4]

(b) Explain why the composite function fg exists and find the corresponding range of fg. [2]

(c) Given that $\mathrm{fg}(x) = 14x + 1 - 2\mathrm{e}^{7x}$, find an expression for $\mathrm{g}(x)$ in terms of x. [2]

(d) If the domain of f is further restricted to $x > k$, state the least value of k for which the function f^{-1} exists. Verify that $\mathrm{f}(1) = 3$ and use this result to find the gradient of the tangent to the curve $y = \mathrm{f}^{-1}(x)$ at $x = 3$. [4]

9 The plane p_1 contains the point $A(-4, 4, 0)$ and the line with equation $\dfrac{x-2}{2} = \dfrac{y-4}{3}, z = 6$.

The plane p_2 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$, where λ and μ are real constants.

(a) Show that the line of intersection between p_1 and p_2 is parallel to the vector $\begin{pmatrix} 5 \\ 6 \\ 1 \end{pmatrix}$.

Hence write down the vector equation of the line of intersection between p_1 and p_2. [5]

(b) Determine the acute angle between p_1 and p_2. [2]

(c) Find the position vector of the foot of perpendicular from point A to p_2. [3]

(d) The plane p_3 has equation $ax + 3y + 2z = b$, where $a, b \in \mathbb{R}$. Find the values of a and b such that all three planes have a common line of intersection. [2]

10 An object is moving from rest in a gas chamber and t seconds later, its velocity v metres per second satisfies the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 5 - 0.2v^2.$$

It is given that $\frac{\mathrm{d}v}{\mathrm{d}t} > 0$ and $v \geq 0$ for all values of t.

(a) Find t in terms of v, simplifying your answer. [5]

(b) Describe how the velocity of the object varies with time. [3]

(c) The displacement of the object, in metres, at any time m seconds is given by $\int_0^m v \, \mathrm{d}t$.

By evaluating this integral, show that the displacement of the object is $5\ln\left(\frac{\mathrm{e}^m + \mathrm{e}^{-m}}{2}\right)$. [4]

11 [It is given that the volume of a circular cone with base radius r and vertical height H is $\frac{1}{3}\pi r^2 H$.]

Intravenous (IV) fluids are specially formulated liquids that are injected into one of the veins of patients for medical treatment. The fluids are contained in a bag connected to an IV tube. The rate of flow of IV fluids into the body is regulated either manually or using an electric pump.

Figure 1 shows an IV fluid bag, modelled by a cylindrical and a conical section. The bag is initially filled with fluid. The radius of the cylinder and the cone is 4 cm. The heights of the cylindrical and conical sections are 20 cm and 4 cm respectively. The height from the apex of the cone to the fluid level is h cm.

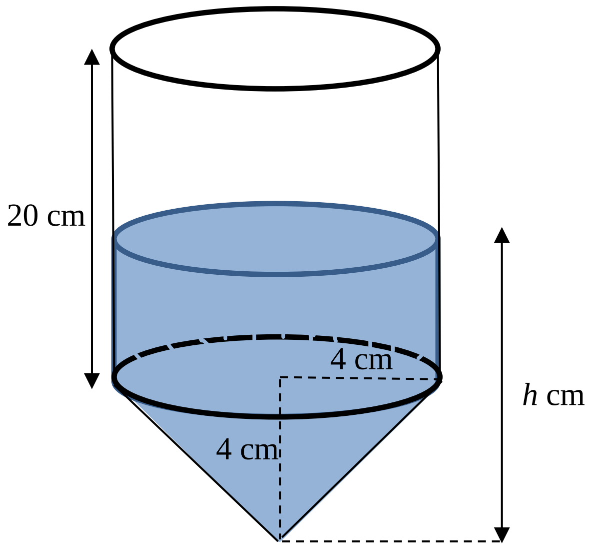


Figure 1

(a) Fluid is to be delivered to the patient at a rate of 50 cm^3 per hour. Determine the value of h and the volume of IV fluid left in the bag when h is decreasing at a rate of 1.21 cm per hour. [5]

Once the IV fluid enters the body, it is mixed with the blood and circulated through the blood vascular system (arteries, capillaries and veins). Due to the viscous nature of blood and friction with the blood vessel walls, resistance to blood flow is generated. To measure this resistance, the blood vessel is modelled as a cylinder with length L and radius r. The resistance R is given by the equation

$$R = \mu\left(\frac{L}{r^4}\right),$$

where μ is a positive constant determined by the viscosity of the blood in the body.

Figure 2 shows a main blood vessel with radius r_1, branching at an acute angle of θ into a smaller blood vessel with radius r_2. The length AD is given as m while the length CD is given as k.

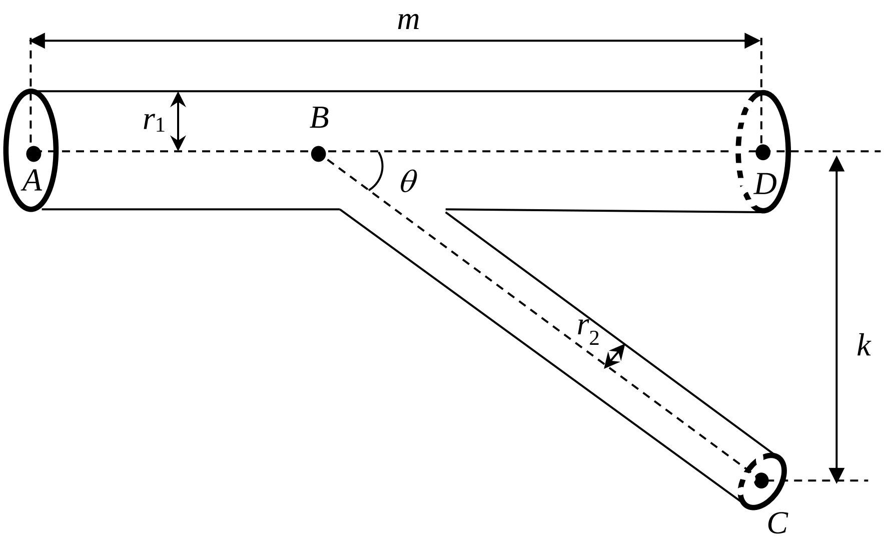


Figure 2

The total resistance, R_T, of the path ABC is the sum of the resistance of the path AB (with radius r_1) and the resistance of the path BC (with radius r_2).

(b) Show that $R_T = \mu\left(\frac{m - k\cot\theta}{r_1^4} + \frac{k\operatorname{cosec}\theta}{r_2^4}\right)$. [2]

(c) As θ varies, show that $\cos\theta = \frac{r_2^4}{r_1^4}$ gives a stationary value of R_T. [3]

(d) Explain, with appropriate working, why the stationary value in part **(c)** provides the least resistance. [2]