1 The geometric series is given by $\frac{3x-2}{4x}+\frac{(3x-2)^2}{4x}+\frac{(3x-2)^3}{4x}+...$ where $x \neq 0$.

Explain why the sum to infinity exists for $\frac{1}{3}<x<1$, and find its value in terms of x. [5]

2 The equation of a curve C is given by

$$y^3+xy-x^2=1.$$

(i) Find the x-coordinate of the point at which the tangent is parallel to the x-axis. [4]

(ii) Show that $\left(3y^2+x\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+6y\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+2\frac{\mathrm{d}y}{\mathrm{d}x}-2=0.$

Given that the value of $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$ when $x=0$ is $\frac{20}{27}$, find the Maclaurin series for y, up to and including the term in x^3. [3]

3 The complex number z is such that $\arg(z)=-\frac{\pi}{3}$ and the complex number w has modulus of 2 and an argument of $\frac{\pi}{4}$. It is also known that $\arg(w-z)=\frac{\pi}{3}$. Let W and Z be the points representing the complex numbers w and z respectively.

(i) By using an Argand diagram with reference from the origin O, show that angle $OWZ=\frac{\pi}{12}$ radians. [2]

(ii) Hence or otherwise, find z and w in the form of $a+b\mathrm{i}$ where a and b are real constants. Leave your answers correct to 3 significant figures. [3]

4 The curve C has the equation $y=ax+\frac{b}{x-2}$, where a and b are non-zero constants.

(i) State the condition(s) required for a and b for this curve to have two turning points. [2]

(ii) Given that $a=b$, find the range of values that y can take, in terms of a. [4]

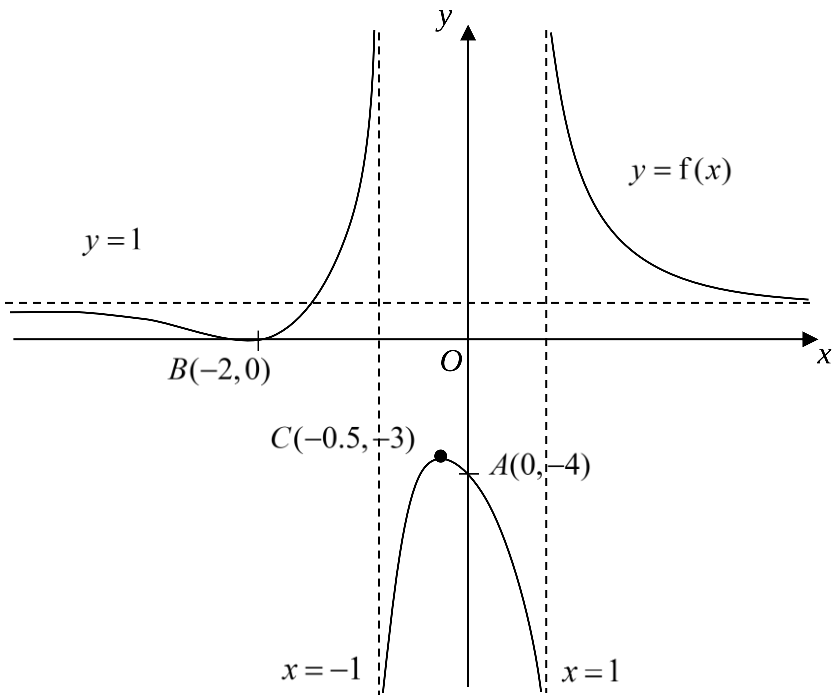
(iii) The inequality

$$ax+\frac{b}{x-2}<ax+m$$

where $a>0$, $b<0$ and m is a positive constant, has the solution set $\left\{x\in\mathbb{R} : x<0 \text{ or } x>2\right\}$.

Find m in terms of b. [2]

5 The diagram below shows the graph of $y = \mathrm{f}(x)$. It cuts the y–axis at $A(0,-4)$, has a minimum point at $B(-2,0)$ and a maximum point at $C(-0.5,-3)$. The equations of the horizontal asymptote is $y = 1$ and vertical asymptotes are $x = -1$ and $x = 1$ respectively. Moreover, when $x = 0$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -3$.



Sketch, on separate diagrams, the graphs of

(i) $y = \mathrm{f}(|x-1|)$, [3]

(ii) $y = \dfrac{1}{\mathrm{f}(x)}$, [3]

(iii) $y = \mathrm{f}'(x)$, [3]

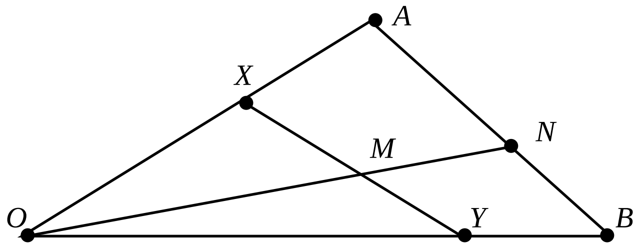
stating clearly the equations of all the asymptotes, coordinates of any points of intersection with both axes and the coordinates of the points corresponding to *A*, *B* and *C* (if any and when possible).

6 **(a)** With reference to the origin O, the point P has coordinates (p_1, p_2, p_3) and the point Q has coordinates (q_1, q_2, q_3).

By considering $\overrightarrow{OP} \cdot \overrightarrow{OQ}$, show that

$$(p_1q_1 + p_2q_2 + p_3q_3)^2 \le (p_1^{\,2} + p_2^{\,2} + p_3^{\,2})(q_1^{\,2} + q_2^{\,2} + q_3^{\,2}).$$ [2]

(b)



With reference to the origin O, the points A and B are such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
It is given that $\overrightarrow{OX} = \frac{2}{3}\mathbf{a}$, $\overrightarrow{OY} = \frac{3}{4}\mathbf{b}$ and the line ON bisects the line XY at the point M.

(i) Find the vector $\overrightarrow{OM}$ in terms of **a** and **b**. [1]

(ii) Find the ratio $AN : NB$. [4]

7 **(a)** Find $\int \frac{4}{1+\cos 4x}\,dx$. [3]

(b) Use integration by parts to show that

$$\int (x-\pi)\sin 2x\,dx = -\frac{1}{2}(x-\pi)\cos 2x + \frac{1}{4}\sin 2x + c,$$

where c is an arbitrary constant.

Hence use the result to obtain the exact value of $\int_0^{\frac{3\pi}{4}} (x-\pi)\left|\sin 2x\right|\,dx$. [7]

8 **(i)** Show that $\frac{2x+9}{x(x+2)(x+3)} = \frac{3}{2}\left(\frac{1}{x} - \frac{1}{x+2}\right) + \left(\frac{1}{x+3} - \frac{1}{x+2}\right)$. [1]

(ii) Using the method of differences, show that

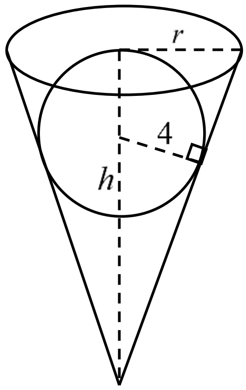
$$\sum_{r=1}^{n} \frac{2r+9}{r(r+2)(r+3)} = \frac{23}{12} - \frac{3}{2}\left(\frac{1}{n+1}\right) - \frac{3}{2}\left(\frac{1}{n+2}\right) + \frac{1}{n+3}.$$ [4]

(iii) Explain why $\sum_{r=1}^{\infty} \frac{2r+9}{r(r+2)(r+3)}$ is convergent and find its exact value. [2]

(iv) Using the result in part **(ii)**, deduce $\sum_{r=3}^{n} \frac{2r+7}{(r+2)(r^2-1)}$. [3]

9 [It is given that a sphere of radius R has surface area $4\pi R^2$ and volume $\frac{4}{3}\pi R^3$, and a cone of radius r and height h has volume $\frac{1}{3}\pi r^2 h$.]

An ice cream company produces ice cream in the shape of a sphere with fixed radius 4 cm, placed in a right circular waffle cone with top radius r cm and height h cm. The ice cream is in contact with the sides of the cone, and the top of the ice cream is at the same level as the opening of the cone (see diagram). Assume the cone is of negligible thickness.



(i) Show that $r = \dfrac{4h}{\sqrt{h^2 - 8h}}$. [2]

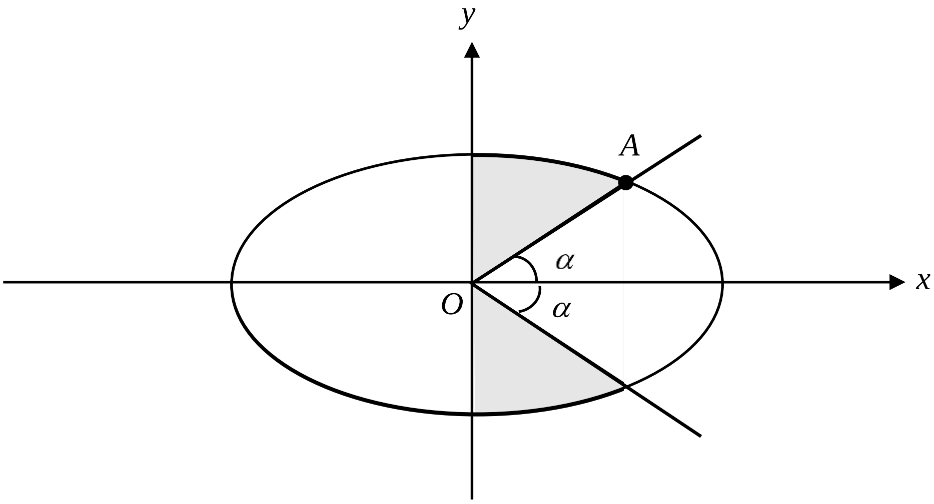
(ii) To improve customer satisfaction, the volume of the empty space inside the cone should be as small as possible, thus the volume of the cone should be minimised. Using differentiation, find the exact minimum volume of the cone. [5]

(iii) When the ice cream is taken out of the freezer, the ice cream starts to melt, while remaining spherical. Given that the surface area of the ice cream is decreasing at a constant rate of $\frac{1}{16}\pi\ \text{cm}^2/\text{min}$, find the rate of decrease of the volume of the ice cream 4 minutes after it had been taken out of the freezer. [5]

10 **(i)** Use the substitution $x = 3\sin\theta$ to show that

$$\int_0^{x_0} \sqrt{9-x^2}\ \mathrm{d}x = \frac{x_0}{2}\sqrt{9-\left(x_0\right)^2} + \frac{9}{2}\sin^{-1}\left(\frac{x_0}{3}\right), \text{ where } 0 \le x_0 \le 3. \qquad [4]$$

The shaded region in the diagram below is enclosed by the curve $\frac{x^2}{9}+\frac{y^2}{4}=1$ for $x \ge 0$, the y-axis and the two lines starting from the origin O, each making an angle α with the x-axis.



Given that $\tan\alpha = \frac{2}{3}$ and A is the point of intersection between the line in the first quadrant and the curve, show that the coordinates of A is $\left(\frac{3}{\sqrt{2}}, \sqrt{2}\right)$. [2]

(ii) Show that the area of the shaded region is $k\pi$, where k is a constant to be determined. [3]

A toy is made by rotating the shaded region completely about the y-axis.

(iii) Find the volume of the toy. [3]

(iv) The toy is now fully enclosed into a cylindrical shape container. State the smallest possible dimensions of the container, assuming that it is made of a material of negligible thickness. [1]

11 A curve is defined by the parametric equations

$$x = \frac{u}{u-1}, \quad y = \frac{u^2}{u-1}, \quad \text{for } u \neq 1.$$

(i) Find the equation of the tangent to the curve at the point with parameter u. [3]

Two distinct points P and Q on the curve have parameters p and q respectively.

(ii) If the tangents at P and Q have the same y-intercept, show that $p+q=0$. [3]

(iii) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x(x-2)}{(x-1)^2}$. Hence find the range of values of x for which the curve is concave upwards. [4]

(iv) Given that P and Q lie on the line $y=5$, find the values of p and q, where $p<q$. Show that the area of the region bounded by the curve and the line $y=5$, is given by $\int_p^q \left(5-\frac{u^2}{(u-1)}\right)\left(\frac{1}{(u-1)^2}\right)\mathrm{d}u$. Hence find this area, giving your answer correct to 2 decimal places. [4]