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ANDERSON SECONDARY SCHOOL
Preliminary Examination 2020
Secondary Four Express & Five Normal

CANDIDATE NAME:

CLASS: / INDEX NUMBER:

ADDITIONAL MATHEMATICS

4047/01

Paper 1

4 August 2020

2 hours

0800 – 1000h

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid/tape.

Answer **all** the questions.
Omission of essential working will result in loss of marks.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the test, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

Setter: Mr Wong Teck Hock

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + ... + \binom{n}{r} a^{n-r}b^r + ... + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ ...\ (n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 Given that θ is obtuse and $\tan\theta = a$, express, in terms of a,

(i) $\cos\theta$, [3]

(ii) $\operatorname{cosec}\theta$. [2]

2 Find the set of values of the constant k for which the curve $y = -x^2 + (1-k)x - 2$ lies entirely below the line $x + y = 0$. [4]

3 Given that $y = he^x + \frac{k}{e^{2x}}$, and that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2\frac{\mathrm{d}y}{\mathrm{d}x} = e^x + 2e^{-2x}$, find the value of each of the constants h and k. [4]

4 A cylindrical ice block of base radius r cm is melting in such a way that the total surface area, A cm^2, is decreasing at a constant rate of 72 cm^2/s. Given that the height is twice the radius and assuming that the ice block retains its shape, calculate the rate of change of r when $r = 5$. [4]

5 Ms Lee bought her dream car last year. The value, $A, of the car is given by the formula $A = 150\,000e^{-pt}$, where p is a constant and t is the age of the car in months. The value of the car after 2 years is expected to be $120 000.

(i) Find the amount which Ms Lee paid for the car. [1]

(ii) Determine the value of the car after 40 months. Give your answer to the nearest dollar. [3]

(iii) Find the age of the car when its value drops to $60 000. Give your answer to the nearest month. [2]

6 **(i)** Factorise $2x^3 + 3x^2 - 8x - 12$. [3]

(ii) Hence solve the equation $\frac{x^3}{4} + \frac{3}{4}x^2 - 4x - 12 = 0$.. [3]

7 **(i)** Write down and simplify the first three terms in the expansion, in descending powers of x, of $\left(1-\frac{2}{x}\right)^8$. [2]

(ii) Given that there is no x term in the expansion of $\left(1-2x-kx^2\right)\left(1-\frac{2}{x}\right)^8$, find the constant term in the expansion. [4]

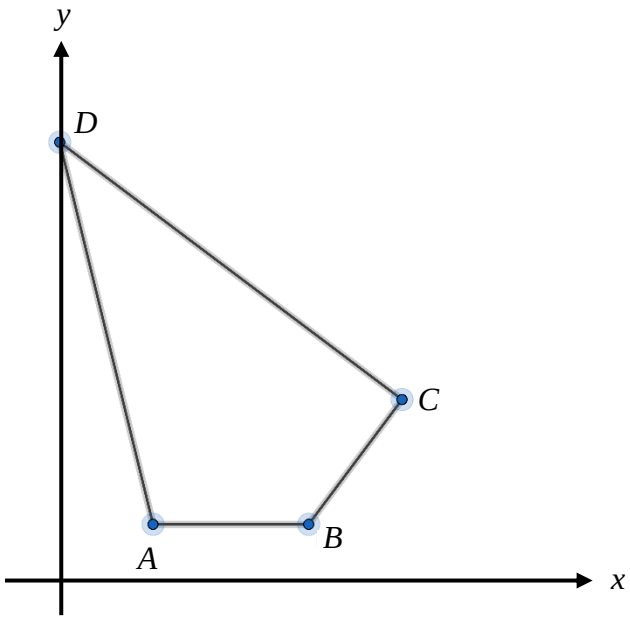
8 The equation of a curve is $y = 2 - x - \frac{2x+3}{x-3}$.

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

(ii) Find the x-coordinate(s) of the stationary point(s) of the curve. [3]

(iii) Determine the nature of each stationary point. [2]

9



The diagram shows a quadrilateral $ABCD$ in which $AB = BC$, $\angle BCD = 90°$ and D is a point on the y-axis. The coordinates of the points A, B and C are $(3, 2), (8, 2)$ and $(11, k)$ respectively.

(i) Given that $k > 2$, show that $k = 6$. [3]

(ii) Find the coordinates of D. [3]

(iii) Find the length of CD and hence find the area of the quadrilateral $ABCD$. [5]

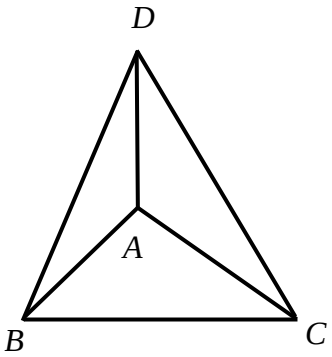
10 **(a)** Find the values of x and y which satisfy the equations

$$\frac{1}{\sqrt{e^{2x-4y}}} = \frac{\sqrt[3]{e}}{e},$$

$$\frac{10^y}{2^x} = 2\left(5^{x+1}\right).$$

[5]

(b)



A solid triangular pyramid *ABCD*, with base *ABC* and vertex *D* such that *D* is vertically above *A*, has a base area of $\left(8+2\sqrt{5}\right)$ cm^2 and height $\left(12-\sqrt{5}\right)$cm .

The top part of the pyramid is removed by a cut parallel to its base and passing through the midpoint of *AD*. Find the volume of the remaining solid, leaving your answer in the form $\frac{7\left(a+b\sqrt{5}\right)}{12}$cm^3, where *a* and *b* are integers. [4]

11 A particle moving in a straight line passes a fixed point O with a velocity 6 ms^{-1}. The acceleration of the particle, a ms^{-2}, is given by $a = 2t - 5$, where t seconds is the time after passing O. Find

(i) the values of t when the particle is instantaneously at rest, [3]

(ii) the displacement of the particle from O at $t = 3$, [3]

(iii) the total distance travelled by the particle in the first 3 seconds of its motion. [2]

12 It is given that $\mathrm{f}(x) = 2\sin\frac{x}{2}$ and $\mathrm{g}(x) = 3\cos x + 1$ where $0 \le x \le 2\pi$.

(i) State the period of $\mathrm{f}(x)$. [1]

(ii) State the smallest value of $\mathrm{f}(x)$. [1]

(iii) State the largest value of $\mathrm{g}(x)$. [1]

(iv) State the largest value of $|\mathrm{f}(x) - \mathrm{g}(x)|$. [1]

(v) Sketch, on the same axes, the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$ for $0 \le x \le 2\pi$. [4]

(vi) Given that the solutions to the equation $\mathrm{f}(x) = \mathrm{g}(x)$ for $0 \le x \le 2\pi$ are a and b where $a < b$, state the range of value of x for which $\mathrm{f}(x) \ge \mathrm{g}(x)$. [1]

End of Paper

Additional Mathematics (4047) Paper 1 - ANSWERS

1 **(i)** $\cos\theta = -\dfrac{1}{\sqrt{1+a^2}}$

(ii) $\text{cosec }\theta = -\dfrac{\sqrt{1+a^2}}{a}$

2 $2-2\sqrt{2} < k < 2+2\sqrt{2}$

3 $h=-1, k=\dfrac{1}{4}$

4 $\dfrac{6}{5\pi}$ cm/s

5 **(i)** \$150 000
(ii) \$103413
(iii) 99 months

6 **(i)** $(2x+3)(x+2)(x-2)$

(ii) $x=-3$, $x=-4$ or $x=4$

7 **(i)** $1-\dfrac{16}{x}+\dfrac{112}{x^2}+.....$

(ii) Constant term $=19$

8 **(i)** $\dfrac{dy}{dx} = -1+\dfrac{9}{(x-3)^2}$,

$\dfrac{d^2y}{dx^2} = -\dfrac{18}{(x-3)^3}$

(ii) 0 and 6

(iii) $\dfrac{d^2y}{dx^2} = \dfrac{2}{3} > 0$, minimum point at $x=0$

$\dfrac{d^2y}{dx^2} = -\dfrac{2}{3} < 0$, minimum point at $x=6$

9 **(i)** $k=6$

(ii) $\left(0, 14\dfrac{1}{4}\right)$

(iii) 65 units^2

10 **(a)** $x=-2\dfrac{2}{3}, y=-1\dfrac{2}{3}$

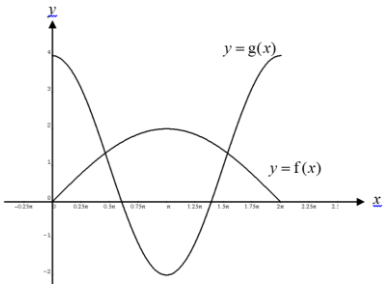
(b) $\dfrac{7\left(43+8\sqrt{5}\right)}{12}$ cm^3

11 **(i)** $t=2$ and $t=3$

(ii) $4\dfrac{1}{2}$ m

(iii) $4\dfrac{5}{6}$ m

12 **(i)** 4π
(ii) 0
(iii) 4
(iv) 4
(v)



(vi) $a \le x \le b$