



**ANGLO-CHINESE JUNIOR COLLEGE
JC2 PRELIMINARY EXAMINATION**

Higher 2

/100

CANDIDATE
NAME

TUTORIAL/
FORM CLASS

INDEX
NUMBER

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MATHEMATICS

9758/02

Paper 2

26 August 2022

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your index number, class and name on all the work you hand in.
Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Write your answers in the spaces provided in the question paper.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved graphing calculator is expected, where appropriate.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 100.

| Question | Marks |
|----------|-------|
| 1 | /7 |
| 2 | /7 |
| 3 | /7 |
| 4 | /8 |
| 5 | /11 |
| 6 | /7 |
| 7 | /10 |
| 8 | /8 |
| 9 | /9 |
| 10 | /13 |
| 11 | /13 |

This document consists of **24** printed pages and **2** blank pages.



Section A: Pure Mathematics [40 marks]

- 1 The function f is defined by

$$f(x) = 1 - ax, \quad x \in \mathbb{R}, \text{ where } a \text{ is a real constant.}$$

The function g is defined by

$$g(x) = \begin{cases} x^2 + 3 & \text{for } 0 \leq x \leq 2, \\ 7 - x & \text{for } 2 < x \leq 7. \end{cases}$$

- (i) Find the set of possible values of a such that f^{-1} exist. [1]

- (ii) If $a = 2$, describe a sequence of transformations that transform the graph of $y = g(x)$ onto the graph of $y = fg(x)$. [3]

- (iii) The function gg is defined by

$$gg(x) = \begin{cases} h_1(x) & \text{for } 0 \leq x \leq 2, \\ h_2(x) & \text{for } 2 < x < 5, \\ h_3(x) & \text{for } 5 \leq x \leq 7. \end{cases}$$

Find $h_1(x)$, $h_2(x)$ and $h_3(x)$. [3]

- 2 It is given that $f(x) = \frac{x+a}{b} - \frac{a}{x+b}$ where $a > b > 0$.

- (i) Sketch the curve with equation $y = f(x)$ and state the equations of any asymptotes and the points where the curve crosses the axes in terms of a and b . [3]

- (ii) Hence or otherwise, solve the inequality $\frac{x+a}{b} \geq \frac{a}{x+b}$. [1]

- (iii) Hence solve the inequality $\frac{a-|x|}{b} \geq \frac{a}{b-|x|}$. [3]

- 3 (i) Find the roots of the equation $iz^2 - (5+i)z + 2 - 6i = 0$, giving your answers in cartesian form $a + bi$, where $a, b \in \mathbb{R}$. [2]

- (ii) Hence find the roots of the equation $-iw^2 - (1-5i)w + 2 - 6i = 0$, giving your answers in cartesian form $a + bi$, where $a, b \in \mathbb{R}$. [2]

(iii) Given that the roots found in part (i) are also roots of the equation $P(z) = 0$, where $P(z)$ is a polynomial of degree 4 with real coefficients, find $P(z)$. [3]

4 (i) It is given that $U_n = \cos[(2n+1)\theta]$, for $n \geq 0$.

Show that for $n \geq 1$, $U_n + U_{n-1} = 2\cos(2n\theta)\cos\theta$. [1]

(ii) Hence show that

$$\sum_{n=1}^{2N} [(-1)^{n+1} \cos(2n\theta)] = \frac{1}{2} \left(1 - \frac{\cos[(4N+1)\theta]}{\cos\theta} \right). \quad [3]$$

(iii) Without the use of the graphic calculator, find the value of $\sum_{n=1}^{41} \left[(-1)^{n+1} \cos \frac{n\pi}{3} \right]$, showing your working clearly. [4]

5 A curve C has parametric equations

$$x = e^{\cos^{-1} 2t}, \quad y = (1 - 4t^2)^{\frac{1}{2}}, \quad \text{where } -\frac{1}{2} < t < \frac{1}{2}.$$

(i) Find $\frac{dy}{dx}$ in terms of t . What can be said about the tangent to C at $t = 0$? [3]

(ii) Sketch the curve C , stating the coordinates of any axial intercepts. [1]

(iii) Find the equation of the tangent to C at the point $P\left(e^{\frac{\pi}{3}}, \frac{\sqrt{3}}{2}\right)$. [2]

(iv) Find the equation of the normal at the point Q on C with parameter q which is parallel to the y -axis. [2]

(v) Find the area bounded by C , the tangent to C at point P and normal to C at point Q . [3]

Section B: Probability and Statistics [60 marks]

- 6 Cathy has 14 magnets of which 5 are red, 4 are blue, 3 are orange and 2 are green.
- (i) Assuming that the magnets of the same colour are identical, find the number of ways in which Cathy can choose 3 magnets. [2]
- (ii) The table below show the amount of money Cathy paid for each type of magnets. Cathy did not pay for the green magnets as they were given to her as free gifts.

| Colour of magnet | Red | Blue | Orange | Green |
|-----------------------|-----|------|--------|-------|
| Price paid per magnet | \$3 | \$1 | \$2 | Free |

- Cathy decided to randomly choose 4 magnets without replacement. Find the probability that she chooses \$7 worth of magnets. [3]
- (iii) Cathy decided to label all the magnets such that each magnet will be distinct from the others. If Cathy were to arrange these 14 magnets in a circle on the whiteboard, find the number of different arrangements such that the 2 green magnets are adjacent to each other and the 3 orange magnets are separated from each other. [2]

- 7 Archer and Betty took part in a competition comprising of at most 3 games. Each game is either won by Archer or Betty. The first person who win 2 games wins the competition. The probability of Archer winning the first game is 0.25. The probability of him winning any subsequent games is p and is independent of any previous games.

(i) Draw a probability tree diagram to represent the above information. [1]

(ii) Find, in terms of p , the probability that Archer will win the competition. [2]

For the rest of the question, use $p = 0.5$.

(iii) Find the probability that Betty won the second game, given that she won the competition. [3]

The number of games won by Archer in a competition is denoted by W . Using $p = 0.5$,

(iv) determine the probability distribution of W , [2]

(v) find $\text{Var}(W)$. [2]

- 8 Cheddar cheese quality is influenced by starter cultures, milk composition and age. Cheddar takes about two to eighteen months to ripen and develop its texture and flavour. The ages in months (m) and prices in dollars (P) of a random sample of ten 1-kilogram Cheddar cheese are given in the table.

| | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|------|------|------|------|
| m | 2.2 | 2.8 | 5.4 | 6.5 | 8.8 | 9.2 | 10.5 | 12.4 | 16.8 | 17.2 |
| P | 25 | 22 | 28 | 32 | 36 | 50 | 72 | 95 | 188 | 240 |

It is thought that the price after m months can be modelled by one of the formulae

$$P = am + b, \quad \ln P = cm + d,$$

where a , b , c and d are constants.

- (i) Explain the meaning of the value of a in the context of the data for the model $P = am + b$. [1]
- (ii) Find, correct to 4 decimal places, the value of the product moment correlation coefficient between
 (A) m and P
 (B) m and $\ln P$ [2]
- (iii) Explain which of the two models in part (ii) is the better model and find the equation of a suitable regression line for this model. [2]
- (iv) Use the equation of the regression line found in (iii), estimate the price of a 1-kilogram Cheddar cheese when it has been aged for 14 months, leaving your answer to the nearest cent. [1]
 Explain whether you would expect this value to be reliable. [1]
- (v) Re-write your equation from part (iii) so that it can be used when the price of the Cheddar cheese, P , is given in dollar **per gram**. [1]
- 9 In this question you should state clearly all the distributions that you use, together with the values of the appropriate parameters. You should also assume that T and X follow independent normal distributions.

Each KTX train takes T minutes to travel from Seoul Train Station to Pohang Train Station. It is known that T follows the distribution $N(144, 25)$.

- (i) The probability that a randomly selected KTX train takes more than k hours to reach Pohang is 0.5. Without the use of a calculator, explain why $k = 2.4$. [1]

Before 7pm daily, each express bus takes X minutes to travel from Seoul Express Bus Terminal to Pohang Express Bus Terminal. It is known that X follows the distribution

$N(236, 81)$. After 7pm daily, the travel time taken by each express bus will be reduced by 10% as all the express buses will not make a stop at Daegu.

- (ii) Find the probability that after 7pm, to travel from Seoul to Pohang, the travel time of a randomly selected KTX train is at most an hour faster than the travel time of a randomly selected express bus. [4]

Kim needs to travel from Seoul to Pohang on Saturday mornings to visit his parents. He prefers to take a KTX train if the tickets are available. On average, 70% of his journeys are by train.

- (iii) On a particular Saturday morning, there was a train delay of 1.6 hours due to a train fault. Given that Kim took more than 4 hours to reach Pohang on that morning, find the probability that Kim travels to Pohang by train. [3]

The cost of taking a KTX train and taking an express bus from Seoul to Pohang are \$54 and \$24 respectively.

- (iv) Find Kim's expected cost of travelling (one-way) from Seoul to Pohang. [1]

- 10** A nasi lemak stall holder uses fresh chicken wings as an ingredient for fried chicken wings. Based on his past years records, his mean daily profit was \$535. With the recent lack of fresh chicken supply, the stall holder substituted fresh chicken wings with frozen chicken wings as the ingredient. His wife was hesitant to the change and claimed that the mean daily profit will decrease. To test his wife's claim, the stall holder takes a random sample of 45 days and recorded the daily profits, x .

- (i) State appropriate hypotheses to test the wife's claim and define any symbols that you use. [2]
- (ii) State, with a reason, whether it is necessary to assume that his past years records of daily profits are normally distributed for the test to be valid. [1]
- (iii) Based on the past years records, it is assumed that the population variance of the daily profit is 2591. If the test shows that there is sufficient evidence that the wife's claim is accepted at 5% level of significance, determine the set of possible values of \bar{x} , the mean daily profit in the 45 days. [2]
- (iv) The stall holder found that $\bar{x} = \$520$ and suspects that the population variance of 2591 may be incorrect. Hence he decided to use the sample variance value of 2008 to test his wife's claim. State the conclusion of the test, showing your workings clearly. [3]
- (v) State the largest significance level that the stall holder should use so that the conclusion in (iv) will be different. Leave your answer in 2 decimal places. [1]

The stall holder now suspects that the mean daily profit does not differ from \$535, even if frozen chicken wings are used instead of fresh chicken wings. To test his claim, the stall holder decides to increase the number of randomly selected days, n , to record his daily profit.

- (vi) It is given that $\bar{x} = 526$ and the population variance is assumed to be 2591. Determine the greatest value of n , so that the conclusion of the test shows that there is no reason to reject the null hypothesis at 8% level of significance. [4]

- 11(a)**
- (i) It is known that the probability of a customer using e-payment at a hawker stall is 0.25. A group of customers is chosen at random, find the probability that the 7th chosen customer is the 5th customer using e-payment at the hawker stall. [3]
- (ii) A sample of 40 customers were randomly chosen from the hawker stall each day. In a month of 30 days, find the probability that there is at least 15 days with at most 10 customers per sample making e-payment. [3]
- (b) The probability that a hawker uses the online delivery platform Foodgowhere is p . A random sample of n hawkers is taken and the random variable X denotes the number of hawkers in the sample that uses Foodgowhere.
- (i) Explain what is meant by a random sample in this context. [1]
Assuming that X follows a binomial distribution.
- (ii) It is given that $P(X \leq 1) = 0.05303$ and the expected number of hawkers using Foodgowhere is 3.96. Write down two equations satisfied by p and n . Hence find the value of p and n . [3]
- (iii) Given that $n = 15$, find the set of values of p so that that the most likely number of hawkers in the sample who uses Foodgowehere is 5. [3]

| Summary of Areas for Improvement | | | |
|----------------------------------|-----------------------|-------------------------------|------------------|
| Knowledge (K) | Careless Mistakes (C) | Read/Interpret Qn wrongly (R) | Presentation (P) |
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