RAFFLES INSTITUTION
2020 YEAR 6 PRELIMINARY EXAMINATION

CANDIDATE NAME

CLASS 20

MATHEMATICS 9758/01

PAPER 1

3 hours

Candidates answer on the Question Paper
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

FOR EXAMINER'S USE						
Q1	**Q2**	**Q3**	**Q4**	**Q5**	**Q6**	**Q7**
/4	**/4**	**/5**	**/6**	**/7**	**/9**	**/10**
Q8	**Q9**	**Q10**	**Q11**	**Q12**	**Total**	
/10	**/10**	**/11**	**/12**	**/12**	**/100**	

This document consists of **28** printed pages and **0** blank page.

RAFFLES INSTITUTION
Mathematics Department

1 A particle is moving along the curve with equation $16x^2+9y^2=144$.
$\frac{\mathrm{d}y}{\mathrm{d}t}=2\,\text{cm s}^{-1}$ and $\frac{\mathrm{d}x}{\mathrm{d}t}$ is positive when $x=\sqrt{5}$.
Find the rate of increase of x at this instant. [4]

2 Mr. Li invested a total of \$30000 and divided this sum into three accounts, which paid 2%, 3% and 5% annual interest respectively.

At the end of the first year, Mr. Li withdrew all the money out from the 2% and 5% accounts and gave the interest earned to his son. The amount in the 3% account, including interest, was re-invested in the same account for another year.

At the end of the second year, Mr. Li withdrew all the money out of the 3% account and gave the interest earned to his son.

The total interest received by the son from the three accounts was \$1423.50.

Given that the amount invested in the 2% account was \$1000 more than the amount invested in the 5% account, find the amounts invested in each of the three accounts. [4]

3 **(i)** Prove that for $x>0$, the substitution $y=ux$ reduces the differential equation $(y-x)\left(\frac{\mathrm{d}y}{\mathrm{d}x}-\frac{y}{x}\right)=y^2+2x^2$ to

$$\left(\frac{u}{u^2+2}-\frac{1}{u^2+2}\right)\left(\frac{\mathrm{d}u}{\mathrm{d}x}\right)=1.$$ [2]

(ii) Hence find the general solution of the differential equation

$$(y-x)\left(\frac{\mathrm{d}y}{\mathrm{d}x}-\frac{y}{x}\right)=y^2+2x^2 \text{ for } x>0.$$ [3]

4 **(i)** Sketch the curve with equation $x^2+y^2-6x=7$. [2]

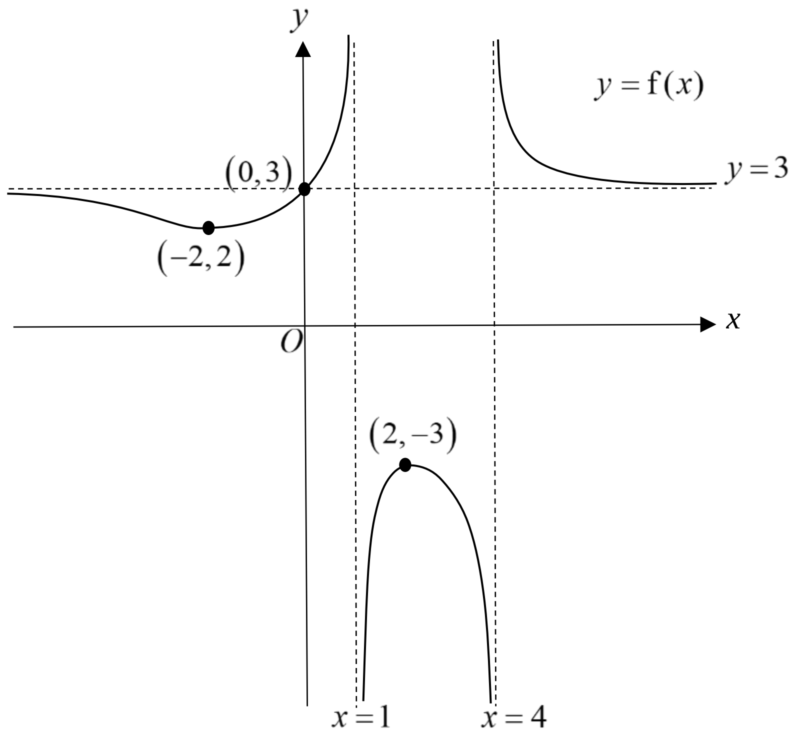
(ii) The region R is bounded by the curve $x^2+y^2-6x=7$, for $x\geq 3$, and the line $x=3$. Given that $\int_0^a \sqrt{a^2-x^2}\,\mathrm{d}x=\frac{\pi a^2}{4}$, find the exact volume of the solid of revolution formed when R is rotated completely about the y-axis. [4]

5 **(a)** Find $\int x\cos x^2 \, dx$. [1]

(b) Use integration by parts to find $\int x\cos 2x \, dx$. [3]

Hence or otherwise find $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x\cos^2 x \, dx$. [3]

6 The diagram below shows the curve of $y = f(x)$. The curve has a minimum point at $(-2, 2)$, a maximum point at $(2, -3)$ and cuts the y-axis at $(0, 3)$. The lines $x = 1$, $x = 4$ and $y = 3$ are the asymptotes to the curve.



On separate diagrams, draw sketches of the following graphs, stating the exact coordinates of any turning points and/or points of intersection with the axes, and the equations of any asymptotes, where possible.

(a) $y = f(1-x)$. [3]

(b) $y = \dfrac{1}{f(x)}$. [3]

(c) $y = f'(x)$. [3]

7 It is given that $\mathrm{f}\left(r\right)=\dfrac{2^r}{r-2}$. Show that $\mathrm{f}\left(r+2\right)-\mathrm{f}\left(r\right)=\dfrac{\left(3r-8\right)2^r}{r\left(r-2\right)}$. [2]

(i) Show that $\displaystyle\sum_{r=3}^{n}\frac{\left(3r-8\right)2^r}{r\left(r-2\right)}=\frac{(3n-2)2^{n+1}}{n(n-1)}-16$. [4]

(ii) Hence find $\displaystyle\sum_{r=1}^{n}\frac{\left(3r-2\right)2^r}{r\left(r+2\right)}$ in the form $\dfrac{(3n+A)2^{n+1}}{(n+B)(n+1)}+C$ where A, B and C are integers to be determined. [4]

8 **Do not use a calculator in answering this question.**

(a) **(i)** Solve the equation $z^2=4\mathrm{i}-3$. [3]

(ii) Solve the equation $z^4+6z^2+25=0$. [3]

(b) Find the modulus and argument of the complex number $w=\dfrac{8-2\mathrm{i}}{5+3\mathrm{i}}$. Hence find the possible values of the positive integer n for which w^n is real. [4]

9 A curve C has parametric equations

$$x=t\ln t,\qquad y=\frac{4}{\mathrm{e}^t}+\mathrm{e}^t,$$

for $t\geq\dfrac{1}{2}$.

(i) C meets the y-axis at point P and line L is the normal to C at P. Show that the equation of L is

$$y=\frac{\mathrm{e}}{4-\mathrm{e}^2}x+\frac{4+\mathrm{e}^2}{\mathrm{e}}.$$ [4]

(ii) Sketch the curve C, stating the coordinates of any turning points and points of intersection with the axes. [3]

(iii) The finite region bounded by C, L and the line $x=\dfrac{1}{2}\ln\left(\dfrac{1}{2}\right)$ is denoted by R. Find the area of R. [3]

10 The plane π_1 contains the point $A(1,2,-1)$ and the line l with equation $\frac{x-1}{2}=\frac{2-z}{3}$, $y=-1$. The plane π_2 contains the point $B(-5.5,3,2)$ and meets π_1 in the line l.

(i) Find the equation of π_1 in scalar product form. [3]

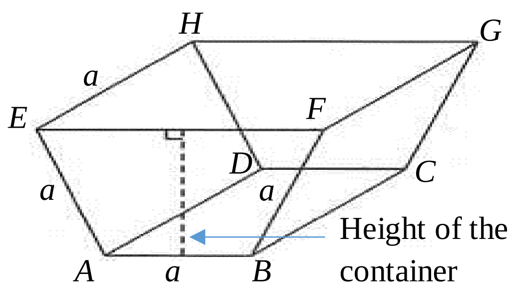
(ii) Show that the vector $\overrightarrow{BF}$ is $\begin{pmatrix} 4.5 \\ -4 \\ 3 \end{pmatrix}$, where F is the foot of perpendicular from B to l. [3]

(iii) Find the exact value of the shortest distance from B to π_1. [2]

(iv) Hence or otherwise find the acute angle between π_1 and π_2, giving your answer to the nearest $0.1°$. [3]

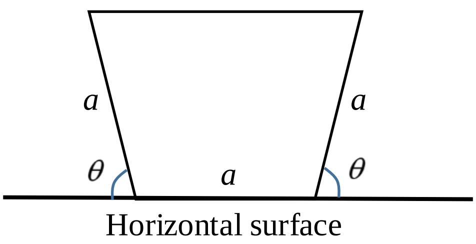
11 Figure 1 shows an open container in the form of a trapezoidal prism *ABCDEFGH* with square base *ABCD* and $AB = AE = BF = EH = a$ cm, where a is a constant. The container is made of plastic of negligible thickness and is placed on a horizontal surface. The faces *BCGF* and *ADHE* are inclined at an angle θ radians, $0<\theta<\frac{\pi}{2}$, to the horizontal surface, and faces *ABFE* and *DCGH* are perpendicular to base *ABCD*. Figure 2 shows its cross-sectional view.

Figure 1



Trapezoidal Prism

Figure 2



Cross-Sectional View

(i) Show that the volume V cm^3 of the container is given by $V = a^3 \sin\theta(1+\cos\theta)$. [2]

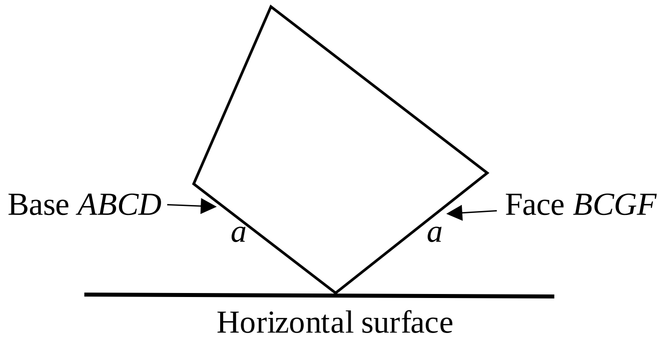
(ii) Use differentiation to find, in terms of a, the maximum value of V in exact form, proving that it is a maximum. [5]

(iii) A particular container is constructed with $\theta = \dfrac{\pi}{3}$ and it is filled with water to half its height. Find, in terms of a, the exact volume of water in this container. [3]

The container is then tilted in the direction of the face $BCGF$ until face $BCGF$ and base $ABCD$ makes the same angle with the horizontal surface.

Figure 3 shows its cross-sectional view.

Figure 3



Cross-Sectional View

Explain if it is possible to tilt the container to this position without any water flowing out from the container. [2]

12 The von Bertalanffy growth model, introduced in 1938, is widely used in fisheries studies. It is used to predict the length, L mm of a fish over a period of time, t years. If L_∞ is the maximum length for a species, then the model assumes that the rate of growth in length of a fish is proportional to $L_\infty - L$.

(i) By setting up and solving a differential equation, show that the general solution of this differential equation is given by $L = L_\infty - Ae^{-kt}$, where k is the constant of proportionality and A is a positive constant. [5]

For the species of fish known as the Atlantic croaker, it has been determined that $L_\infty = 419$ mm and at one year of age, its length is 219 mm and the rate of growth in length is 55 mm per year. Using the above model, obtain an expression for L in terms of t. [3]

(ii) Find its age when the Atlantic croaker grows to a length of 300 mm. [2]

(iii) Sketch a graph of L against t. [2]