TEMASEK JUNIOR COLLEGE,
SINGAPORE
JC 2
Preliminary Examination 2019

CANDIDATE NAME | | CG |

MATHEMATICS

9758/02

Higher 2

16 Sep 2019

Paper 2

3 hours

Candidates answer on the Question Paper.
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your Civics group and name on all the work that you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiner's Use	
Question Number	Marks Obtained
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total Marks	

This document consists of **24** printed pages and **no** blank pages.

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1 The function f is defined by

$$\mathrm{f}: x \mapsto \frac{x^2}{2-x},\ x \in \mathbb{R},\ 0 \le x < 2\ .$$

(i) Find $\mathrm{f}^{-1}(x)$ and write down the domain of f^{-1}. [4]

It is given that

$$\mathrm{g}: x \mapsto \frac{1}{1+\mathrm{e}^{-x}},\ x \in \mathbb{R},\ x \ge 0\,.$$

(ii) Show that fg exists. [2]

(iii) Find the range of fg. [2]

2 Express $\dfrac{6r+7}{r(r+1)}$ as partial fractions. [1]

(i) Hence use method of differences to find $\displaystyle\sum_{r=1}^{N}\left(\left(\frac{1}{7}\right)^{r+1}\frac{6r+7}{r(r+1)}\right)$ in terms of N.

(There is no need to express your answer as a single algebraic fraction.) [3]

(ii) Give a reason why the series $\displaystyle\sum_{r=1}^{\infty}\left(\left(\frac{1}{7}\right)^{r+1}\frac{6r+7}{r(r+1)}\right)$ converges, and write down its value. [2]

(iii) Use your answer in part **(i)** to find $\displaystyle\sum_{r=1}^{N}\left(\left(\frac{1}{7}\right)^{r+1}\frac{6r+13}{(r+1)(r+2)}\right)$. [3]

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3 A curve C with equation $y = \mathrm{f}(x)$ satisfies the equation

$$(x^2 + 2x + 2)\frac{\mathrm{d}y}{\mathrm{d}x} = 2$$

and passes through the point $(0, \pi)$.

(i) By further differentiation, find the Maclaurin expansion of $\mathrm{f}(x)$ in ascending powers of x up to and including the term x^3. [5]

(ii) Solve the differential equation $(x^2 + 2x + 2)\frac{\mathrm{d}y}{\mathrm{d}x} = 2$, given that $y = \pi$ when $x = 0$, leaving y in terms of x.

Hence show that

$$\tan^{-1}(x+1) \approx \frac{\pi}{4} + \frac{1}{2}x - \frac{1}{4}x^2 + \frac{1}{12}x^3$$

for small values of x. [4]

(iii) With the aid of a sketch, explain why $\int_0^2 \frac{\pi}{4} + \frac{1}{2}x - \frac{1}{4}x^2 + \frac{1}{12}x^3 \, \mathrm{d}x$ gives a more accurate approximation of $\int_0^2 \tan^{-1}(x+1) \, \mathrm{d}x$ than $\int_0^2 \frac{\pi}{4} + \frac{1}{2}x - \frac{1}{4}x^2 \, \mathrm{d}x$. [2]

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4 The points A, B, C and D have coordinates $(1, 0, 3)$, $(-1, 0, 1)$, $(1, 1, 3)$ and $(1, k, 0)$ respectively, where k is a positive real number. The plane p_1 contains A, B and C while the plane p_2 contains A, B and D.

Given that p_1 makes an angle of $\frac{\pi}{3}$ with p_2, show that $k = \frac{\sqrt{6}}{2}$. [5]

The point X lies on p_2 such that the vector $\overrightarrow{XC}$ is perpendicular to p_1. Find $\overrightarrow{XC}$. [5]

Hence find the exact area of the triangle AXC. [2]

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Section B: Probability and Statistics [60 marks]

5 Anand, Beng, Charlie, Dayanah and 6 other people attend a banquet dinner, and are to sit at a round table.

(i) Dayanah will only sit next to Anand, Beng or Charlie (and no one else), and Anand, Beng and Charlie do not want to sit next to each other. Find the number of ways the 10 people can seat themselves around the table. [4]

(ii) As part of dinner entertainment, 4 people from the table are chosen to participate in a game.

Among Anand, Beng and Charlie, if any one of them is chosen, the other two will refuse to participate in the game. Furthermore, Dayanah refuses to participate unless at least one of Anand, Beng or Charlie is also chosen.

Find the number of ways the 4 people can be chosen for the game. [3]

6 A bag contains four identical counters labelled with the digits 0, 1, 2, and 3. In a game, Amira chooses one counter randomly from the bag and then tosses a fair coin. If the coin shows a Head, her score in the game is the digit labelled on the counter chosen. If the coin shows a Tail, her score in the game is the negative of the digit labelled on the counter chosen. T denotes the score in a game.

(i) Find the probability distribution of T. [2]

(ii) Amira tosses the coin and it shows a Tail. Find the probability that $T < -1$. [3]

(iii) Amira plays the game twice. Find the probability that the sum of her two scores is positive. [3]

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7 It is generally accepted that a person's diet and cardiorespiratory fitness affects his cholesterol levels. The results of a study on the relationship between the cholesterol levels, C mmol/L, and cardiorespiratory fitness, F, measured in suitable units, on 8 individuals with similar diets are given in the following table.

Cardiorespiratory Fitness (F units)	55.0	50.7	45.3	40.2	34.7	31.9	27.9	26.0
Cholesterol (C mmol/L)	4.70	4.98	5.30	5.64	6.04	6.30	6.99	6.79

(i) Draw a scatter diagram of these data. Suppose that the relationship between F and C is modelled by an equation of the form $\ln C = aF + b$, where a and b are constants. Use your diagram to explain whether a is positive or negative. [4]

(ii) Find the product moment correlation coefficient between ln C and F, and the constants a and b for the model in part **(i)**. [3]

(iii) Bronz is a fitness instructor. His cardiorespiratory fitness is 52.0 units. Estimate Bronz's cholesterol level using the model in **(i)** and the values of a and b in part **(ii)**. Comment on the reliability of the estimate. [2]

(iv) Bronz then had a medical checkup and found his actual cholesterol level to be 6.2 mmol/L. Assuming his cholesterol level is measured accurately, explain why there is a great difference between Bronz's cholesterol level and the estimated value in **(iii)**. [1]

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8 A research laboratory uses a data probe to collect data for its experiments. There is a probability of 0.04 that the probe will give an incorrect reading. In a particular experiment, the probe is used to take 80 readings, and X denotes the number of times the probe gives an incorrect reading.

(i) State, in context, two assumptions necessary for X to be well modelled by a binomial distribution. [2]

(ii) Find the probability that between 5 and 10 (inclusive) incorrect readings are obtained in the experiment. [3]

When the probe gives an incorrect reading, it will give a reading that is 5% greater than the actual value.

(iii) Suppose the 80 readings are multiplied together to obtain a Calculated Value. Find the probability that the Calculated Value is at least 50% more than the product of the 80 actual values. [5]

9 A Wheel Set refers to a set of wheel rim and tyre. The three types of wheel sets are the Clincher Bike Wheel Set, Tubular Bike Wheel Set and Mountain Bike Wheel Set.
The weight of a rim of a Clincher Bike Wheel Set follows a normal distribution with mean 1.5 kg and standard deviation 0.01 kg. The weight of its tyre follows a normal distribution with mean 110 g and standard deviation 5 g.

(i) Let C be the total weight in grams of a randomly chosen Clincher Bike Wheel Set in grams. Find $\mathrm{P}(C > 1620)$. [3]

(ii) State, in the context of the question, an assumption required in your calculation in **(i)**. [1]

Let T be the total weight in grams of a Tubular Bike Wheel Set, where $T \sim \mathrm{N}(\mu,\ 15^2)$.

(iii) The probability that the weight of a randomly chosen Clincher Bike Wheel Set exceeds a randomly chosen Tubular Bike Wheel Set by more than 150 g is smaller than 0.70351 correct to 5 decimal places. Find the range of values that μ can take. [5]

Let M be the total weight in grams of a randomly chosen Mountain Bike Wheel Set with mean 1800 g and standard deviation 20 g.

(iv) Find the probability that the mean weight of 50 randomly chosen Mountain Bike Wheel Sets is more than 1795 g. [3]

10 **(a)** Two random samples of different sample sizes of households in the town of Aimek were taken to find out the mean number of computers per household there. The first sample of 50 households gave the following results.

Number of computers	0	1	2	3	4
Number of households	5	12	18	10	5

The results of the second sample of 60 households were summarised as follows.

$$\sum y = 118 \qquad \sum y^2 = 314,$$

where y is the number of computers in a household.

(i) By combining the two samples, find unbiased estimates of the population mean and variance of the number of computers per household in the town. [4]

(ii) Describe what you understand by 'population' in the context of this question. [1]

(b) Past data has shown that the working hours of teachers in a city are normally distributed with mean 48 hours per week. In a recent study, a large random sample of n teachers in the city was surveyed and the number of working hours per week was recorded. The sample mean was 46 hours and the sample variance was 131.1 hours^2. A hypothesis test is carried out to determine whether the mean working hours per week of teachers has been reduced.

(i) State appropriate hypotheses for the test. [1]

The calculated value of the test statistic is $z = -1.78133$ correct to 5 decimal places.

(ii) Deduce the conclusion of the test at the 2.5 % level of significance. [2]

(iii) Find the value of n. [3]

(iv) In another test, using the same sample, there is significant evidence at the α% level that there is a change in the mean working hours per week of teachers in that city. Find the smallest possible integral value of α. [2]

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