TEMASEK JUNIOR COLLEGE,
SINGAPORE
JC 2
Preliminary Examination 2019

CANDIDATE NAME ______ CG ______

MATHEMATICS

9758/01

Higher 2

30 Aug 2019

Paper 1

3 hours

Candidates answer on the Question Paper.
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your Civics group and name on all the work that you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiner's Use	
Question Number	Marks Obtained
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
Total Marks	

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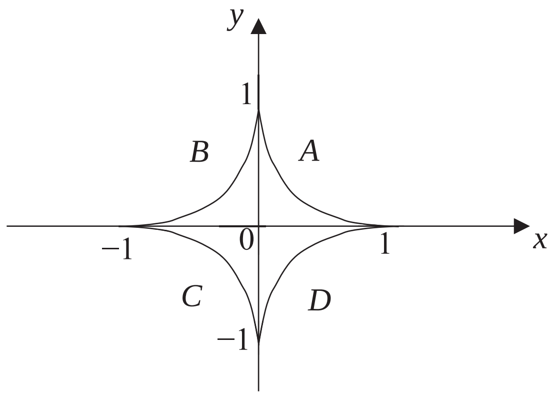
1 The diagram below shows a shape which is symmetrical about the x- and y-axes. The shape is made up of four curves, A, B, C and D.



The curve A has equation $\sqrt{x}+\sqrt{y}=1$ for $0 \le x \le 1$ and $0 \le y \le 1$.

(i) State the equations and the range of values of x and y for curves B and C. [3]

(ii) The curves A and B are scaled by a factor $\frac{1}{2}$ parallel to the x-axis and the curves C and D are scaled by a factor 2 parallel to the y-axis. Sketch the resulting shape. [2]

2 The position vectors of A, B, C and D are $\begin{pmatrix}\alpha\\1\\5\end{pmatrix}$, $\begin{pmatrix}1\\-2\\1\end{pmatrix}$, $\begin{pmatrix}-2\\3\\1\end{pmatrix}$ and $\begin{pmatrix}1\\\beta\\7\end{pmatrix}$ respectively, where α and β are real numbers. Given that BD is a perpendicular bisector of AC, find the values of α and β. [5]

3 The hyperbola C passes through the point (2, 0) and has oblique asymptotes $y=-2x$ and $y=2x$.

(i) Sketch C, showing the relevant features of the curve. [2]

(ii) Write down the equation of C. [1]

(iii) By adding a suitable curve to your sketch in part **(i)**, solve the inequality

$$\sqrt{\frac{x^2}{4}-1} < \sqrt{x-1}\,. \qquad [3]$$

4 A curve C has equation $y = \dfrac{e^x}{x-k}$, $x \neq k$, where k is a positive real number.

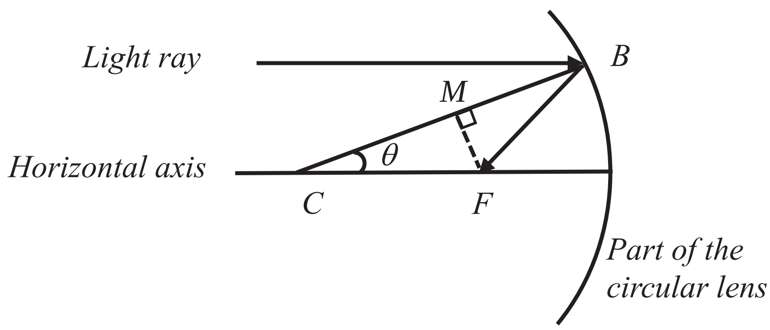
Show algebraically that C has exactly one stationary point, and show that the stationary point lies in the first quadrant. [3]

Sketch C for $x > k$, indicating clearly the equation of the asymptote and the coordinates of the stationary point. [2]

Deduce that $\displaystyle\int_{k+\frac{1}{2}}^{k+\frac{3}{2}} \frac{e^x}{x-k}\,dx < \frac{1}{3}\left(3e^{k+\frac{1}{2}} + e^{k+\frac{3}{2}}\right)$ for all positive real values of k. [2]

5 In geometric optics, the paraxial approximation is a small-angle approximation used in Gaussian optics and ray tracing of light through an optical system such as a lens.

In the diagram below, a light ray parallel to the horizontal axis is reflected at point B on the circular lens centred at point C and has radius r cm. Let $\angle BCF = \theta$ radians. FM is the perpendicular bisector of CB.



(i) Show that $CF = \dfrac{r}{k\cos\theta}$, where k is a real constant to be determined. [1]

(ii) Hence find the series expansion for CF if θ is sufficiently small for θ^3 and terms in higher powers of θ to be neglected. [2]

Suppose that the source of the light ray is now repositioned such that $\angle BCF = \left(\theta + \dfrac{\pi}{6}\right)$ radians.

(iii) Find the corresponding series expansion for CF, up to and including the term in θ^2. [4]

6 An arithmetic sequence has first term a and common difference d, where a and d are non-zero. The ninth, tenth and thirteenth terms of the arithmetic sequence are the first three terms of a geometric sequence.

(i) Show that $a = -\frac{15}{2}d$. [3]

(ii) The sum of the first n terms of the arithmetic sequence is denoted by S_n. Find the value of S_{16}. [2]

(iii) Given that the k^{th} term of the arithmetic sequence is the fourth term of the geometric sequence, find the value of k. [3]

7 A curve C has parametric equations

$$x = t^2, \qquad y = t\,\mathrm{e}^{t^2}, \qquad \text{for } t \geq 0.$$

(i) Find the equation of the tangent to C at the point P with coordinates $\left(p^2, p\,\mathrm{e}^{p^2}\right)$, where $p \neq 0$. Hence, or otherwise, find the exact equation of the tangent L to C which passes through the origin. [5]

(ii) **(a)** Find the cartesian equation of C. [1]

(b) Find the exact volume of the solid formed when the region bounded by C and L is rotated through 2π radians about the x-axis. [5]

8 **Do not use a calculator in answering this question.**

The complex numbers z and w are given by $z = \dfrac{(1+\mathrm{i})^4}{(1-\mathrm{i})^2}$ and $w = \dfrac{8}{\left(\sqrt{3}+\mathrm{i}\right)^2}$.

(i) Express z and w in polar form $r(\cos\theta + \mathrm{i}\sin\theta)$, where $r > 0$ and $-\pi < \theta \leq \pi$. Give r and θ in exact form. [4]

(ii) Given that z^2, w and w^* are the roots of the equation $x^3 + bx^2 + cx + d = 0$ where b, c and d are real values, find the equation. [3]

(iii) Sketch on an Argand diagram with origin O, the points P, Q and R representing the complex numbers z, w and $z + w$ respectively. [2]

(iv) By considering the quadrilateral $OPRQ$ and the argument of $z + w$, deduce that

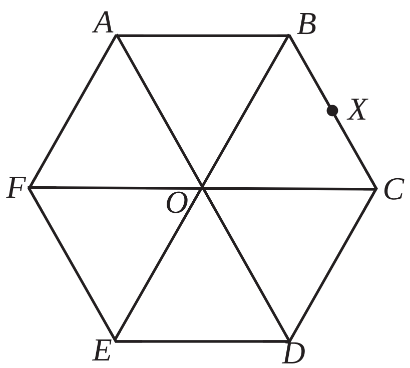
$$\tan\frac{5\pi}{12} = 2 + \sqrt{3}.$$ [3]

9 **(a)** Vectors $\mathbf{u}$ and $\mathbf{v}$ are such that $\mathbf{u}.\mathbf{v} = -1$ and $(\mathbf{u} \times \mathbf{v}) + \mathbf{u}$ is perpendicular to $(\mathbf{u} \times \mathbf{v}) + \mathbf{v}$.

Show that $|\mathbf{u} \times \mathbf{v}| = 1$. [3]

Hence find the angle between $\mathbf{u}$ and $\mathbf{v}$. [3]

(b) The figure shows a regular hexagon *ABCDEF* with *O* at the centre of the hexagon. *X* is the midpoint of *BC*.



Given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$, find $\overrightarrow{OF}$ and $\overrightarrow{OX}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [2]

Line segments *AC* and *FX* intersect at the point *Y*. Determine the ratio *AY* : *YC*. [4]

10 Mr Ng wants to hang a decoration on the vertical wall above his bookshelf. He needs a ladder to climb up.

The rectangle $ABCD$ is the side-view of the bookshelf and HK is the side-view of the ladder where $AB = 24$ cm and $BC = 192$ cm (see **Figure 1**). The ladder touches the wall at H, the edge of the top of the bookshelf at B and the floor at K.

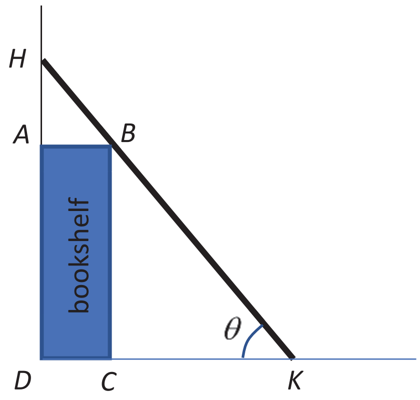


Figure 1

(i) Given that $\angle HKD = \theta$, show that the length, L cm of the ladder is given by

$$L = \frac{24}{\cos\theta} + \frac{192}{\sin\theta} .$$ [1]

(ii) Use differentiation to find the exact value of the shortest length of the ladder as θ varies. [4]
[You do not need to verify that this length of the ladder is the shortest.]

Take L to be 270 for the rest of this question.

The ladder starts to slide such that H moves away from the wall and K moves towards E (see **Figure 2**). The ladder maintains contact with the bookshelf at B.

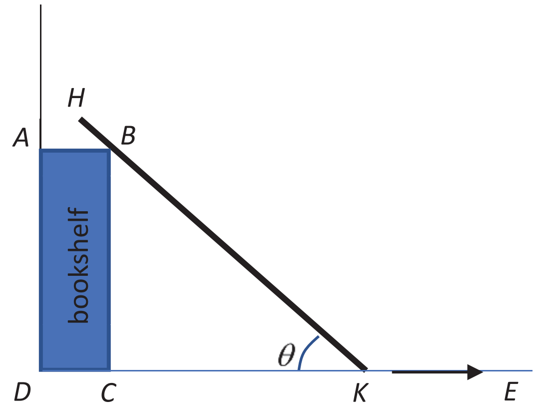


Figure 2

The horizontal distances from the wall to H and from the wall to K are x cm and y cm respectively.

(iii) By expressing $y - x$ in terms of θ, determine whether the rate of change of y is greater than the rate of change of x. [3]

(iv) Given that the rate of change of θ is -0.1 rad s^{-1} when $CK = 160$ cm, find the rate of change of x at this instant. [5]

11 The daily food calories, L, taken in by a human body are partly used to fulfill the daily requirements of the body. The daily requirements is proportional to the body mass, M kg, with a constant of proportionality p. The rate of change in body mass is proportional to the remaining calories.

It is given that the body mass, M kg, at time t days satisfies the differential equation

$$\frac{\mathrm{d}M}{\mathrm{d}t} = k\left(L - pM\right),$$

where k and L are constants.

John's initial body mass is 100 kg. Find, in terms of p, the daily food calories needed to keep his body mass constant at 100 kg. [1]

To lose weight, John decides to start on a diet where his daily food calorie intake is 75% of the daily calories needed to keep his body mass constant at 100 kg.

(i) Show that $M = 75 + 25\mathrm{e}^{-pkt}$. [4]

(ii) John attained a body mass of 90 kg after 50 days on this diet. If it takes him n more days to lose at least another 10 kg, find the smallest integer value of n. [5]

(iii) John's goal with this diet plan is to achieve a body mass of 70 kg. With the aid of a graph, explain why he can never achieve his goal. [2]

(iv) By considering $\frac{\mathrm{d}^2M}{\mathrm{d}t^2}$, comment on his rate of body mass loss as time passes. [2]