NANYANG JUNIOR COLLEGE

JC2 PRELIMINARY EXAMINATION

Higher 2

CANDIDATE NAME														
CT CLASS	1	9			Centre Number/ Index Number					/				

MATHEMATICS **9758/02**

Paper 2 **15 September 2020**

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and class on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

For examiner's use only	
Question number	Marks
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
Total	

This document consists of **6** printed pages and **0** blank page.

Section A: Pure Mathematics [40 marks]

1 In mathematics, the polar coordinate system is a two-dimensional coordinate system in which each point on a plane is determined by a distance from a reference point and an angle from a reference direction. It provides a method of rendering graphs and indicating the positions of points on a two-dimensional surface. The polar coordinate system is employed in mathematics, physics, engineering, navigation, robotics, and other sciences.

The area of a region enclosed by a curve with polar equation $r = \mathrm{f}(\theta)$ where $\alpha \le \theta \le \beta$, is given by the integral $\frac{1}{2}\int_{\alpha}^{\beta} [\mathrm{f}(\theta)]^2 \, \mathrm{d}\theta$.

A curve C has polar equation $r = a\left(\cos 2\theta - \sin\theta\right)$, where a is a positive constant and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{6}$. By using the formulae from the List of Formulae (MF26), show that the area of the region enclosed by C is given by $\frac{1}{4}a^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{6}} \left(\cos 4\theta - \cos 2\theta - 4\cos 2\theta \sin\theta + 2\right) \, \mathrm{d}\theta$. [2]

Hence find, in terms of a, the exact area enclosed by C. [4]

2 The function f is given by $\mathrm{f} : x \mapsto x^2 + 2kx$, for $x \in \mathbb{R}, x \ge -k$.

(i) Find $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1} in terms of k. [3]

For the rest of this question, let $k = -2$.

(ii) The function g is given by

$$\mathrm{g} : x \mapsto 5 + 7\sqrt{1 - \frac{(x+1)^2}{4^2}}, \text{ for } x \in \mathbb{R},\ -5 \le x \le 3.$$

If fg(x) = 32, find the exact value(s) of x. [4]

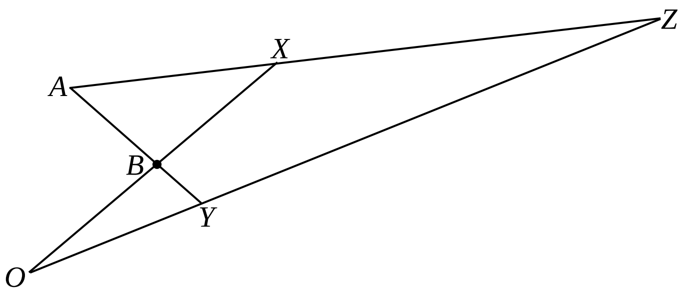
3 A curve C has parametric equations

$$x = \left(1 - 4\theta^2\right)^{\frac{1}{2}},\ y = \cos^{-1}(2\theta), \text{ where } -\frac{1}{2} \le \theta \le 0.$$

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of θ. Hence find the exact coordinates of the point on C such that the normal to the curve at this point is parallel to the line $y - x = 2$. [4]

(ii) Q is a point moving on C such that its y-coordinate increases at a rate of 0.25 units per second. Find the value of θ at the point Q when the rate of change of the gradient at Q is 1 unit per second. [4]

4



Referred to the origin O, points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively. Points X and Y are such that B is the mid-point of OX and B lies between A and Y with $AB : BY = 2 : 1$. The lines AX and OY meet at the point Z.

(i) Find the position vectors $\overrightarrow{OX}$, $\overrightarrow{OY}$ and $\overrightarrow{OZ}$ giving your answers in terms of $\mathbf{a}$ and $\mathbf{b}$. [6]

(ii) Given that $OA = 2\sqrt{7}$, $OB = 3$ and the area of triangle OAZ is $\sqrt{1701}$, find the acute angle between OA and OB, giving your answer exactly. [3]

5 **(a)** One of the roots of the equation $x^3 + px^2 + 6x + q = 0$, where p and q are real, is $5 - \mathrm{i}$. Find the other roots of the equation and the values of p and q. [4]

(b) **Do not use a calculator in answering this part.**

(i) The complex number z has modulus $2\sqrt{3}$ and argument $-\frac{\pi}{3}$. Find $z - 2\sqrt{3}$ in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \le \pi$. Give r and θ in exact form. [3]

(ii) Find the three smallest positive whole number values of n for which $\dfrac{\left(z-2\sqrt{3}\right)^*}{\left(z-2\sqrt{3}\right)^n}$ is a real number. [3]

Section B: Probability and Statistics [60 marks]

6 **(a)** Find the number of ways of arranging all the eight letters in the word ATTITUDE if the first and last letter must be a consonant and the T's are separated. [3]

(b) An organisation has 7 members in the Marketing team and 10 members in the Operations team. A committee consisting of 6 people is to be formed and it has been decided that it must include at least 2 people from the Marketing team and at least 3 people from the Operations team.

(i) Find the number of ways in which the committee can be formed. [2]

The committee of 6 people is then finalised and they join another 3 members from the Finance team to go for a bonding activity. During the activity, these 9 people are required to stand in a circle.

(ii) Find the number of ways to arrange them. [1]

(iii) Find the number of ways to arrange them such that all members of the Finance team are standing next to each other. [2]

7 A jar contains five 20-cent coins and n 50-cent coins, where $n \geq 2$. In a game, Lee removes coins at random from the jar, one at a time, until he has **at least** 70 cents. He scores zero points if he removes only two coins, otherwise his score is **half** the number of coins he removed. The score Lee obtains is denoted by the random variable S.

(i) Show that $\mathrm{P}(S=0)=\dfrac{n(n+9)}{(n+5)(n+4)}$. [1]

(ii) Find P(S = s) for all other possible values of s. [2]

(iii) Show that $\mathrm{E}(S)=\dfrac{30}{(n+5)(n+3)}$ and $\mathrm{Var}(S)=\dfrac{15n\left(3n^2+40n+113\right)}{(n+5)^2(n+4)(n+3)^2}$. [5]

8 A manufacturer produces teacups, saucers and plates. Past records indicate that on average 5% of the teacups, 2% of the saucers and 1% of the plates produced are flawed. The quality of the teacups, saucers and plates are independent of one another. A box contains a teacup, a saucer and a plate. A box is considered imperfect if any of the three items are flawed. A consignment consists of 250 boxes. The number of boxes in a consignment that are imperfect is denoted by X. You may assume that X can be modelled by a binomial distribution.

(i) Show that the probability that a randomly selected box is imperfect is 0.07831. [1]

(ii) Find the probability that there are at least 10 but less than 30 boxes that are imperfect in a consignment. [2]

(iii) Find the least value of r such that the probability that there are more than r boxes that are imperfect in a consignment is at most 0.125. [3]

(iv) Using a suitable approximation, find the probability that the total number of boxes that are imperfect in a sample of 100 consignments is at most 2000. State an assumption that you have made in your calculations. [4]

9 A group of 240 residents who live in NY Gardens are surveyed on whether they like cats, hamsters or dogs as pets. The survey result are as follows:

- 80 residents like hamsters,
- 120 residents like either hamsters or cats (or both),
- 24 residents like both hamsters and dogs.

There are some residents who like both cats and hamsters but no resident like both cats and dogs. Furthermore, no residents like all three animals.

You may assume that a resident's liking for cats and hamsters are independent.

One resident from this group is chosen at random. The events where a resident likes cats, hamsters and dogs are denoted by C, H and D respectively.

(i) Draw a Venn diagram to illustrate clearly the relationship between C, H and D. [1]

(ii) Find $\mathrm{P}(C' \cap H')$. [1]

(iii) Find $\mathrm{P}(C)$. [3]

(iv) Find the greatest value of $\mathrm{P}(D)$, justifying your answer. [2]

(v) If a resident who does not like hamsters has a probability of $\frac{1}{8}$ in liking dogs, find $\mathrm{P}(D)$. [3]

10 **In this question you should state clearly all the distributions that you use, together with the values of the appropriate parameters.**

The Nanyang Symphony Orchestra plays a mixture of classical and jazz music pieces. The length, in minutes, of each piece of music are modelled as having independent normal distributions. The means and standard deviations of these distributions are shown in the following table.

	Mean	Standard deviation
Classical piece	12	2.3
Jazz piece	7	1.5

The orchestra organises a challenge whereby it will be playing a repertoire of 5 classical and 4 jazz pieces which are randomly chosen by a live audience.

(i) Calculate the probability that the total length of playing the 5 randomly chosen classical pieces is less than 50 minutes. [2]

(ii) Find the probability that the mean length of these 9 randomly chosen pieces in the repertoire is less than 8 minutes. [2]

(iii) The orchestra decides to have an interval of exactly one minute between the orchestra finishing one piece and starting the next piece. The challenge starts at 7.00 pm. By using your answer in part **(ii)**, state whether it is likely that the orchestra will still be playing after 8.20 pm. Justify your answer. [2]

The orchestra plans to record the repertoire into an album.

(iv) The orchestra estimates that the total recording time will be **twice** the total length of the 9 pieces in the repertoire and proceeds to book a slot with the recording studio. The booking contract of the recording studio states that if the time taken to record the repertoire exceeds the booked duration of x minutes, a fine will be imposed. Find the minimum duration, to the nearest minute, for which the orchestra must book the recording studio if it wants to be at least 99% sure that it will not be fined. [3]

(v) Due to the different expertise and resources involved in recording classical and jazz pieces, the recording studio charges a rate of \$12 per minute for a classical piece and \$9.50 per minute for a jazz piece. Find the probability that the total charge of recording the 5 randomly chosen classical pieces is within \$300 of the total charge of recording the 4 randomly chosen jazz pieces. [3]

11 The production manager of a food manufacturing company wishes to take a random sample of a certain type of chocolate bars from the thousands produced one day at his factory, for quality control purposes. He wishes to check that the mean mass of the chocolate bars produced by the company is 100 grams, as stated on the packets.

(i) State what it means for a sample to be random in this context. [1]

The masses, x grams, of a random sample of 70 chocolate bars are summarised as follows.

$$n = 70 \qquad \sum(x-95) = 287 \qquad \sum(x-95)^2 = 1928$$

(ii) Calculate unbiased estimates of the population mean and variance of the mass of chocolate bars. [2]

(iii) Carry out the test, at the 5% level of significance, for the production manager. Give your conclusion in context. [4]

(iv) The production manager receives complaints from the customers that there is an overestimation of the mass of the chocolate bars and decides to investigate further. State appropriate null and alternative hypotheses for this investigation. By using the data from the existing sample of 70 chocolate bars, determine, at the 5% level of significance, whether the company has overstated the mean mass of the chocolate bars. [2]

(v) To reduce the number of complaints, the company revises the mean mass of the chocolate bars printed on the packets to m grams, where m is a positive integer. A quality control officer from the company wishes to test whether the mean mass of the chocolate bars is the same as that printed on the packets. Using the data from the existing sample of 70 chocolate bars, find the value of m such that there is no reason for the officer to reject the printed revised mean mass at the 5% level of significance. [3]

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