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CONVENT OF THE HOLY INFANT JESUS SECONDARY
Preliminary Examination in preparation for
the General Certificate of Education Ordinary Level 2020

CANDIDATE NAME	

CLASS		REGISTER NUMBER	

ADDITIONAL MATHEMATICS **4047/02**

Paper 2 **14 September 2020**

Candidates answer on the Question Paper. **2 hours and 30 minutes**
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 The function f is defined by $\mathrm{f}(x) = x\mathrm{e}^{-x} + k\mathrm{e}^{\frac{3}{5}x}$, where k is a constant.
Given that $\mathrm{f}'(0) = 5$, find the value of k. [4]

2 Given that $y = 3^x$, solve the equation $6(3^{-2x}) - 1 = 3^{-x}$. [4]

3 **(i)** By using long division, explain how $x-3$ is a factor of $2x^3-13x^2+24x-9$. [3]

(ii) Express $\dfrac{5x^2-30x+10}{2x^3-13x^2+24x-9}$ as the sum of three partial fractions. [5]

Continuation of working space for question 3(ii)

(iii) Hence find $\int \frac{10x^2 - 60x + 20}{2x^3 - 13x^2 + 24x - 9}\,dx$. [4]

[Turn over

4 **(i)** Differentiate $(3x+1)\ln(3x+1)-3x$ with respect to x. [3]

(ii) Hence evaluate $\int_0^2 \ln(3x+1)^2\,dx$. [4]

5 The third term in the binomial expansion of $\left(x-\frac{3}{x}\right)^n$, where n is a positive integer, is kx^8.

(i) Show that $n=12$. [2]

(ii) Find the value of k. [1]

(iii) Find the coefficient of x^8 in the expansion of $\left(x-\frac{3}{x}\right)^{12}\left(2+x^2\right)$. [4]

[Turn over

6 The roots of the quadratic equation $x^2 - 3x - 2 = 0$ are α and β.

(i) Show that $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = -\dfrac{13}{2}$. [3]

(ii) Find a quadratic equation with roots $\alpha - \dfrac{1}{\alpha}$ and $\beta - \dfrac{1}{\beta}$. [5]

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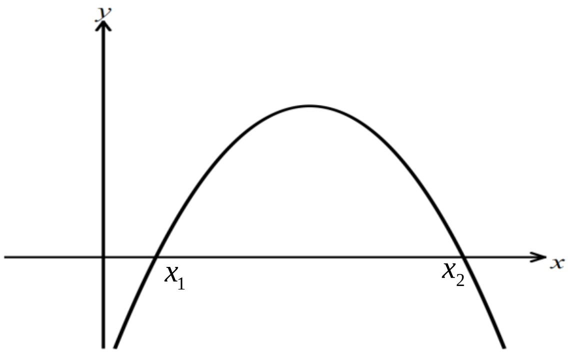


The quadratic curve $y = -x^2 + bx - 1$, where *b* is a positive integer, intersects the *x*-axis at x_1 and x_2 as shown in the diagram.

(i) Explain, showing your working clearly, why the smallest value of *b* is 3. [3]

(ii) Using $b = 3$, find, without the use of a calculator, the exact value of $\frac{x_1}{x_2}$,

leaving your answer in the form $\frac{p + q\sqrt{5}}{r}$, where *p*, *q* and *r* are integers. [4]

[Turn over

8 **(a)** A circle, C_1, has equation $x^2 + y^2 + 4x - 10y - 20 = 0$.

Find the radius and the coordinates of the centre of C_1. [3]

(b) A circle, C_2, passes through the points $A(2,0)$ and $B(8,0)$. The centre of C_2 lies above the x-axis. The y-axis is tangent to C_2.

(i) Show that the y-coordinate of the centre of C_2 is 4 and hence find the equation of the circle, C_2. [4]

Continuation of working space for question 8b(i)

(ii) Find the coordinates of the point on C_2 which is furthest from $A(2, 0)$. [2]

9 **(i)** Show that $4\cos 4x - 3\sin 4x$ can be expressed in the form $R\cos(4x+\alpha)$,
where R and α are constants, $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$. [4]

(ii) Given that $\mathrm{f}(x) = \mathrm{e}^{-3x}\sin 4x$, show that $\mathrm{f}'(x)$ can be written in the form
$\mathrm{f}'(x) = R\mathrm{e}^{-3x}\cos(4x+\alpha)$ where R and α are the constants found in part **(i)**. [3]

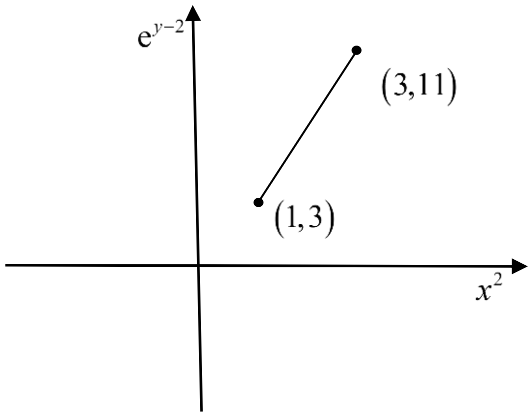
(iii) Hence, find the smallest positive value of x for which the curve $y = \mathrm{f}(x)$ has a stationary point. [3]

10 The point $P(1,-2)$ lies on the curve $y=\mathrm{f}(x)$ such that $\mathrm{f}'(x)=\dfrac{6x^3+9}{x^2}-\dfrac{9}{2}\sqrt{x}$, $x>0$.

(i) Find an expression for $\mathrm{f}(x)$. [4]

(ii) Find the equation of the normal to the curve $y=\mathrm{f}(x)$ at the point $P(1,-2)$ and express it in the form $ax+by+c=0$ where a , b and c are integers. [3]

11 **(a)**



The diagram shows part of a straight line graph obtained by plotting e^{y-2} against x^2.

(i) Given that the line passes through the points $(1,3)$ and $(3,11)$, express y in terms of x. [3]

(ii) Explain clearly why the range of values of x for which the equation found in part **(i)** is not defined for $-\frac{1}{2} \leq x \leq \frac{1}{2}$. [3]

[Turn over

11 **(b)** A new machine is used to measure the surface area of a solid, A cm^2 with a length of x cm. It is known that A and x are related by the equation, $A = px + qx^2$, where p and q are constants. The table below shows corresponding values of A and x. One of the values of A is believed to be inaccurate.

x (cm)	2	4	6	8	10
A (cm^2)	42	148	318	552	700

(i) Draw the graph of $\frac{A}{x}$ plotted against x, using a scale of 1 cm for 1 unit on the x-axis and a scale of 1 cm for 5 units on the $\frac{A}{x}$ axis. [3]

(ii) Use the graph to estimate the value of each of the constants p and q. [3]

(iii) Identify the inaccurate value of A and suggest a reason why this may be inaccurate. [2]

[Turn over

12 The curve $y = \frac{a}{2x+1} + 9x - 5$, where a is a constant, has two stationary points.

(i) Given that $\left(-\frac{1}{6}, y\right)$ is one of the stationary points, find y. [5]

(ii) Find the coordinates of the other stationary point. [3]

(iii) Find an expression for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and hence determine the nature of these stationary points. [3]

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Answers to A Math 2020 Prelim P2

1. $k = \frac{20}{3}$

2. $x = 0.631$

3. (ii) $\frac{5x^2 - 30x + 10}{2x^3 - 13x^2 + 24x - 9} = \frac{-3}{5(2x-1)} + \frac{14}{5(x-3)} - \frac{7}{(x-3)^2}$

   (iii) $\frac{-3}{5}\ln(2x-1) + \frac{28}{5}\ln(x-3) + \frac{14}{(x-3)} + c$

4. (i) $3\ln(3x+1)$ (ii) 5.08

5. (ii) 594 (iii) -4752

6. (ii) $\therefore$ equation is $x^2 - \frac{9}{2}x + 4 = 0$ or $2x^2 - 9x + 8 = 0$

7. (ii) $\frac{7 - 3\sqrt{5}}{2}$

8. (a) Radius = 7 units and centre is $(-2, 5)$

   (b) (i) Equation of C_2 is $(x-5)^2 + (y-4)^2 = 25$ (ii) Coordinates are $(8, 8)$

9. (i) $4\cos 4x - 3\sin 4 = 5\cos(4x + 0.644)$ (ii) $5e^{-3x}\cos(4x + 0.644)$ (iii) $x = 0.232$

10. (i) $\mathrm{f}(x) = 3x^2 - 3x^{\frac{3}{2}} - \frac{9}{x} + 7$ (ii) $2x + 21y + 40 = 0$

11. (a) (i) $y = \ln(4x^2 - 1) + 2$

    (b) (ii) $p = 5$, $q = 8$ (ii) Inaccurate $A = 700$

12. (i) when $x = -\frac{1}{6}$, $y = \frac{2}{\left(2\left(-\frac{1}{6}\right) + 1\right)} + 9\left(-\frac{1}{6}\right) - 5 = -\frac{7}{2}$ (ii) coordinates are $\left(-\frac{5}{6}, -\frac{31}{2}\right)$

    (iii)

$$\frac{d^2y}{dx^2} = 8(2x+1)^{-3}.2$$

$$= \frac{16}{(2x+1)^3}$$

$\left(-\frac{1}{6}, -\frac{7}{2}\right)$ is a minimum point, $\left(-\frac{5}{6}, -\frac{31}{2}\right)$ is a maximum point.